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具有随机扰动和 Markov 切换的中立型耦合神经网络的自适应同步*

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摘要： 研究了具有时变时滞和随机扰动的中立型神经网络的自适应同步问题.随机扰动用 Brown 运动来描述.通过 Lyapunov 稳定性理论,利用了 LMI 分析技巧和矩阵理论,研究了具有随机扰动和 Markov 切换的中立型神经网络的自适应同步,给出并证明了使系统同步的充分条件,得出了具有时变时滞和随机扰动的中立型神经网络的自适应同步的判据.最后,给出数值例子来说明理论结果的有效性.

关　键　词： 中立型神经网络；　随机扰动；　牵制控制；　自适应同步

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引　　言

在过去的几十年中,神经网络的稳定性分析、周期振荡以及混沌行为得到了广泛的研究.因此神经网络可应用于自动控制、信号处理、模式识别和优化计算等领域[1-5].神经网络在实际应用中越来越受欢迎,因为其可以通过大规模的集成或数字电路来实现.由于放大器有限的转换速度以及信号传输过程中出现的拥塞现象等因素,使得时间延迟现象不可避免地存在.这种时滞是导致神经网络不稳定或振荡的原因之一,因此具有时滞的神经网络的稳定性与同步问题引起了许多学者的关注,同时获得了许多的成果[6-7].中立型神经网络是一种特殊类型的时滞神经网络,时滞不仅出现在系统现在的状态,而且出现在系统状态的导数中.因此时滞中立型神经网络的稳定性分析已获得了大量的研究成果[4,7-9].

在生物神经系统中突触的信号传递可被视为一个噪声过程,这种噪声主要源于神经传导物质的释放及一些概率性的因素带来的随机波动.在一般情况下,Gauss 噪声经常被视为在神经网络中所引起的干扰.另外,许多神经网络可能会受到某些现象的影响而出现在结构和参数的突然变化,从而改变子系统之间的相互关联以及突然的环境变化.在这种情况下,神经网络可被视为具有有限模态的系统,这些模态在不同的时间从一个跳变到另一个,这种跳变可以用 Markov 链来描述.因此,带有 Markov 跳变的神经网络的应用十分广泛[10-15].文献[10]研究了具有 Markov 中立型延迟神经网络的驱动响应同步.文献[11]通过使用自适应控制方法,考虑了

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具有 Markov 耦合中立型复杂动力网络的指数同步.

在网络无法自行同步的情况下,已经研究出许多控制技术来驱动网络实现同步,如线性状态反馈控制[16]、脉冲控制[17-18]、自适应控制[19-20]、采样数据控制[21]以及它们之间的组合方法,已被广泛运用于混沌系统.这些方法中多数是通过控制网络的全部节点来实现同步的,但是真实世界的网络是由大量的节点构成的,将控制器添加到大规模网络中的所有节点是非常困难的.为了减少受控节点的数量,引入了牵制控制,其中控制器仅适用于部分节点,通过节点之间的耦合作用,从而控制整个网络[22].另外,为了有效获得适当的控制增益,自适应牵制控制方法涌现了大量的研究成果[23-25].

本文利用 Lyapunov 稳定性理论和自适应牵制控制方法,给出了保证中立型神经网络与 Markov 切换参数自适应同步的准则.最后给出了一个数值例子来说明所提方法的有效性.

1 系统模型及预备知识

$\{r(t),t\geqslant 0\}$ 是在完全概率空间上 $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geqslant 0},P)$ 的一个右连续的 Markov 链,它包含满足一般条件(即当 $\mathcal{F}_0$ 包含 P- 空集时单调递增且右连续)的自然流.

$\{\mathcal{F}_t\}_{t\geqslant 0}$ 在有限状态空间 $S=\{1,2,\cdots,\kappa\}$ 中取值.概率转移矩阵为 $\boldsymbol{\Pi}=(\boldsymbol{\pi})_{\kappa\times\kappa}$:

$$P\{r(t+\delta)=j\,|\,r(t)=i\}=\begin{cases}\gamma_{ii}\delta+o(\delta), & i\neq j,\\ 1+\gamma_{ij}\delta+o(\delta), & i=j,\end{cases}$$

其中, $\delta>0$ 且 $\gamma_{ij}\geqslant 0$ 是 i 到 j 的转移率,如果 $i\neq j$, 则 $\gamma_{ii}=-\sum_{j\neq i}\gamma_{ij}$.为简单起见,令 $r(t)=m,m\in S$.

考虑到如下具有 Markov 的中立型时滞神经网络模型:

$$\begin{aligned}&\mathrm{d}[\boldsymbol{s}(t)-\boldsymbol{D}_m\boldsymbol{s}(t-\tau(t))]=\\&\qquad[-\boldsymbol{C}_m\boldsymbol{s}(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{s}(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{s}(t-\tau(t)))+\boldsymbol{J}]\mathrm{d}t,\end{aligned}\tag{1}$$

其中, $\boldsymbol{s}(t)=[s_1(t),s_2(t),\cdots,s_n(t)]^{\mathrm{T}}\in R^n$ 是与 n 个神经元相关联的状态向量, $\boldsymbol{A}_m\in R^{n\times n}$ 为连接权矩阵, $\boldsymbol{B}_m\in R^{n\times n}$ 为时滞连接权矩阵, $\boldsymbol{C}_m=\mathrm{diag}\{c_1,c_2,\cdots,c_n\}$ 是正定对角矩阵, $\boldsymbol{D}_m\in R^{n\times n}$, $\boldsymbol{f}(\cdot)$ 是神经元激活函数且有界, $\tau(t)$ 是时变时滞且满足

$$0\leqslant\tau(t)\leqslant\tau,\ \dot{\tau}(t)\leqslant\hat{\tau}\leqslant 1,\tag{2}$$

其中, $\tau,\hat{\tau}$ 为常数.

考虑到式(1)中 $\boldsymbol{s}(t)$ 是作为同步动态,研究了具有随机扰动和 Markov 跳变的 n 维中立型神经网络.如下所示的随机延迟差分方程表达了它的动态行为:

$$\begin{aligned}&\mathrm{d}[\boldsymbol{x}_i(t)-\boldsymbol{D}_m\boldsymbol{x}_i(t-\tau(t))]=\\&\quad\Big[-\boldsymbol{C}_m\boldsymbol{x}_i(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{x}_i(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{x}_i(t-\tau(t)))+\boldsymbol{J}+\\&\quad+c\sum_{j=1}^{N}g_{ij}^{m}\boldsymbol{\Gamma}_m\boldsymbol{x}_j(t)+\boldsymbol{R}_i(t)\Big]\mathrm{d}t+\boldsymbol{\sigma}(r(t),\boldsymbol{x}_i(t),\boldsymbol{x}_i(t-\tau(t)))\mathrm{d}\omega(t),\\&\qquad\qquad i=1,2,\cdots,N,\end{aligned}\tag{3}$$

其中, $\boldsymbol{x}(t)=(x_1(t),x_2(t),\cdots,x_n(t))^{\mathrm{T}}\in R^n$ 是第 i 个节点的神经元状态向量,系统参数 $\boldsymbol{A}_m$, $\boldsymbol{B}_m,\boldsymbol{C}_m,\boldsymbol{D}_m$ 与式(1)定义相同, $\boldsymbol{R}_i(t)$ 是控制输入向量, c 是耦合强度, $\boldsymbol{\Gamma}=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$ 是对角矩阵,描述的是在时间 t、两个互连节点 i 和 j 之间的内耦合, $\boldsymbol{e}_i(t)=\boldsymbol{x}_i(t)-\boldsymbol{s}(t)$ 表示节点 i 的状态和状态向量 $\boldsymbol{s}(t)$ 之间的误差, $\boldsymbol{\omega}(t)=(\omega_1(t),\omega_2(t),\cdots,\omega_n(t))^{\mathrm{T}}$ 是定义在完

备概率空间 $(\Omega,\{\mathcal{F}_t\}_{t\geqslant 0},P)$ 上具有自然流 $\{\mathcal{F}_t\}_{t\geqslant 0}$ 的 m 维 Brown 运动, $\boldsymbol{\sigma}$ 是满足 $\boldsymbol{\sigma}:\mathbf{R}_+\times R^n\times R^n\to R^{n\times m}$ 的噪声强度函数, $\boldsymbol{G}_m=(g_{ij}^m)_{N\times N}$ 是外耦合转置矩阵,表示中立型神经网络的耦合强度和拓扑结构,对于 $i=1,2,\cdots,N$, 则

$$g_{ii}=-\sum_{j=1,\,j\neq i}^{N}g_{ij},$$

由矩阵 $\boldsymbol{G}_m$ 的性质能够得到

$$\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}\boldsymbol{x}_j(t)=\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}(\boldsymbol{e}_j(t)+\boldsymbol{s}(t))=\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}\boldsymbol{e}_j(t)+\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}\boldsymbol{s}(t)=\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}\boldsymbol{e}_j(t).$$

由式(3)减去式(1),得到如下的误差系统:

$$\begin{aligned}&\mathrm{d}[\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i(t-\tau(t))]=\\&\quad\Big[-\boldsymbol{C}_m\boldsymbol{e}_i(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{e}_i(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{e}_i(t-\tau(t)))+\\&\quad c\sum_{j=1}^{N}g_{ij}^m\boldsymbol{\Gamma}_m\boldsymbol{e}_j(t)+\boldsymbol{R}_{im}(t)\Big]\mathrm{d}t+\boldsymbol{\sigma}(r(t),\boldsymbol{e}_i(t),\boldsymbol{e}_i(t-\tau(t)))\mathrm{d}\boldsymbol{\omega}(t),\\&\qquad i=1,2,\cdots,N,\end{aligned}\tag{4}$$

其中

$$\begin{aligned}&\boldsymbol{f}(\boldsymbol{e}_i(t))=\boldsymbol{f}(\boldsymbol{x}_i(t))-\boldsymbol{f}(\boldsymbol{s}(t))=\boldsymbol{f}(\boldsymbol{e}_i(t)+\boldsymbol{s}(t))-\boldsymbol{f}(\boldsymbol{s}(t)),\\&\boldsymbol{f}(\boldsymbol{e}_i(t-\tau(t)))=\boldsymbol{f}(\boldsymbol{x}_i(t-\tau(t)))-\boldsymbol{f}(\boldsymbol{s}(t-\tau(t)))=\\&\quad\boldsymbol{f}(\boldsymbol{e}_i(t-\tau(t))+\boldsymbol{s}(t-\tau(t)))-\boldsymbol{f}(\boldsymbol{s}(t-\tau(t))).\end{aligned}$$

为证明我们的主要结果,则需要下面的假设、定义和引理.

假设 1　对于任意 $\boldsymbol{x},\boldsymbol{y}\in R^n$, 存在常数 l_i, 函数 $\boldsymbol{f}(\cdot)$ 满足下面 Lipschitz 条件:

$$\|\boldsymbol{f}(\boldsymbol{x})-\boldsymbol{f}(\boldsymbol{y})\|\leqslant l_i\|\boldsymbol{x}-\boldsymbol{y}\|.$$

令 $\boldsymbol{L}=\mathrm{diag}\{\eta_1,\eta_2,\cdots,\eta_n\}$, 可以从假设 1 中得出结论:

$$\begin{aligned}&\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{LMf}(\boldsymbol{e}(t))=\sum_{i=1}^{n}f_i(\boldsymbol{e}_i(t))\eta_i d_i\boldsymbol{e}_i(t)\geqslant\sum_{i=1}^{n}d_i(f_i(\boldsymbol{e}_i(t)))^2=\\&\quad\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{M}\boldsymbol{f}(\boldsymbol{e}(t)),\end{aligned}$$

其中, $\boldsymbol{M}=\mathrm{diag}\{m_1,m_2,\cdots,m_n\}$ 是任意正对角矩阵.

假设 2　噪声强度矩阵 $\boldsymbol{\sigma}(\cdot,\cdot,\cdot)$ 满足有界条件,对于任意 $\boldsymbol{x},\boldsymbol{y}\in R^n$, 存在两个已知常数 h_1 和 h_2, 满足

$$\begin{aligned}&\mathrm{trace}[\boldsymbol{\sigma}^{\mathrm{T}}(r(t),\boldsymbol{e}(t),\boldsymbol{e}(t-\tau(t)))\boldsymbol{\sigma}(r(t),\boldsymbol{e}(t),\boldsymbol{e}(t-\tau(t)))]\leqslant\\&\quad\boldsymbol{e}^{\mathrm{T}}(t)h_1\boldsymbol{e}(t)+\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))h_2\boldsymbol{e}(t-\tau(t)).\end{aligned}$$

引理 1(广义 Itô 公式)　令 $V\in(R^n\times\mathbf{R}_+\times S;\mathbf{R})$ 且 $0\leqslant\tau_1\leqslant\tau_2$ 是有界停止时间.如果 $V(t,i,\boldsymbol{x}(t))$ 和 $LV(t,i,\boldsymbol{x}(t))$ 在 $t\in[\tau_1,\tau_2]$ 上是有界的,故

$$EV(\tau_2,r(\tau_2),\boldsymbol{x}(\tau_2))=EV(\tau_1,r(\tau_1),\boldsymbol{x}(\tau_1))+E\int_{\tau_1}^{\tau_2}LV(s,r(s),\boldsymbol{x}(s))\mathrm{d}s.$$

引理 2　对于任意的 $\zeta(0)\in L_{\mathcal{F}_0}^2([-\delta_2,0];\mathbf{R})$ 和 $r_0=m\in S$, 若误差系统(3)是稳定的,则系统(1)和系统(2)则是同步的,

$$E\int_0^{\tau}\|\boldsymbol{e}(t;\zeta(0),r_0)\|^2\mathrm{d}t<\infty.$$

为了使中立型神经网络能够实现指数同步,设计一个自适应牵制控制器,控制器被添加到一部分节点上.不失一般性,让前 l 个节点被控制,控制器的设计如下:

$$\boldsymbol{R}_i(t)=\begin{cases}-\boldsymbol{k}_i(t)(\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i(t-\tau(t))), & i=1,2,\cdots,l,\\ \boldsymbol{0}, & i=l+1,l+2,\cdots,N.\end{cases} \tag{5}$$

所以,误差系统(4)被描述如下:

$$\begin{cases}\mathrm{d}[\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i(t-\tau(t))]=\\ \qquad\Big[-\boldsymbol{C}_m\boldsymbol{e}_i(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{e}_i(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{e}_i(t-\tau(t)))+\\ \qquad c\sum_{j=1}^{N}g_{ij}^{m}\boldsymbol{\Gamma}_m\boldsymbol{e}_j(t)+\boldsymbol{R}_{im}(t)\Big]\mathrm{d}t+\\ \qquad\boldsymbol{\sigma}(r(t),\boldsymbol{e}_i(t),\boldsymbol{e}_i(t-\tau(t)))\mathrm{d}\boldsymbol{\omega}(t), \qquad i=1,2,\cdots,l,\\ \mathrm{d}[\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i(t-\tau(t))]=\\ \qquad\Big[-\boldsymbol{C}_m\boldsymbol{e}_i(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{e}_i(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{e}_i(t-\tau(t)))+\\ \qquad c\sum_{j=1}^{N}g_{ij}^{m}\boldsymbol{\Gamma}_m\boldsymbol{e}_j(t)\Big]\mathrm{d}t+\\ \qquad\boldsymbol{\sigma}(r(t),\boldsymbol{e}_i(t),\boldsymbol{e}_i(t-\tau(t)))\mathrm{d}\boldsymbol{\omega}(t), \qquad i=l+1,l+2,\cdots,N.\end{cases} \tag{6}$$

通过使用 Kronecker 内积,系统(4)重新定义为

$$\begin{aligned}&\mathrm{d}[\boldsymbol{e}(t)-(\boldsymbol{I}_N\otimes\boldsymbol{D}_m)\boldsymbol{e}(t-\tau(t))]=\\ &\quad[-(\boldsymbol{I}_N\otimes\boldsymbol{C}_m)\boldsymbol{e}(t)+(\boldsymbol{I}_N\otimes\boldsymbol{A}_m)\boldsymbol{f}(\boldsymbol{e}(t))+(\boldsymbol{I}_N\otimes\boldsymbol{B}_m)\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))+\\ &\quad(c\boldsymbol{G}_m\otimes\boldsymbol{\Gamma}_m)\boldsymbol{e}(t)+\boldsymbol{R}_m]\mathrm{d}t+\boldsymbol{\sigma}(r(t),\boldsymbol{e}(t),\boldsymbol{e}(t-\tau(t)))\mathrm{d}\boldsymbol{\omega}(t).\end{aligned} \tag{7}$$

2 主要结果

设计一个自适应控制器使得中立型耦合神经网络(1)和(3)实现同步,主要结果如下.

定理1 在假设1、2成立的条件下,如果存在常数 $\rho_m>0$, 对称矩阵 $\boldsymbol{P}_m>\boldsymbol{0},\boldsymbol{Q}_1>\boldsymbol{0},\boldsymbol{Q}_2>\boldsymbol{0},\boldsymbol{L}>\boldsymbol{0},\boldsymbol{M}>\boldsymbol{0},\boldsymbol{R}>\boldsymbol{0}$, 使得下列 LMIs 成立:

$$\boldsymbol{P}_m<\rho_m\boldsymbol{I}, \tag{8}$$

$$\boldsymbol{\Omega}=\begin{bmatrix}W_{11} & W_{12} & 0 & W_{14} & W_{15}\\ & W_{22} & 0 & W_{24} & W_{25}\\ & & W_{33} & 0 & 0\\ & & & W_{44} & 0\\ * & & & & W_{55}\end{bmatrix}\leqslant\boldsymbol{0}, \tag{9}$$

其中

$$\begin{aligned}W_{11}&=\boldsymbol{I}_N\otimes\left(-\boldsymbol{P}_m\boldsymbol{C}_m+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\frac{1}{2}\rho_m h_1+\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{P}_\kappa\right)-\\ &\quad\boldsymbol{k}^*\otimes\boldsymbol{P}_m+c\boldsymbol{G}_m\otimes\boldsymbol{P}_m\boldsymbol{\Gamma}_m,\\ W_{12}&=\boldsymbol{I}_N\otimes\left(\frac{1}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{C}_m-\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_\kappa\right)-\frac{c}{2}\boldsymbol{G}_m\otimes\boldsymbol{P}_m\boldsymbol{\Gamma}_m\boldsymbol{D}_m,\\ W_{14}&=\boldsymbol{I}_N\otimes\left(\frac{1}{2}\boldsymbol{P}_m\boldsymbol{A}_m+\boldsymbol{L}\boldsymbol{M}\right),\ W_{15}=\boldsymbol{I}_N\otimes\frac{1}{2}\boldsymbol{P}_m\boldsymbol{B}_m,\\ W_{22}&=\boldsymbol{I}_N\otimes\left(\frac{1}{2}\rho h_2-(1-\hat{\tau})\boldsymbol{Q}_2+\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_\kappa\boldsymbol{D}_m\right)+2\boldsymbol{k}^*\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m,\end{aligned}$$

$$W_{24}=-\boldsymbol{I}_N\otimes\frac{1}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{A}_m,\ W_{25}=-\boldsymbol{I}_N\otimes\frac{1}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{B}_m,$$

$$W_{33}=-\boldsymbol{I}_N\otimes\boldsymbol{Q}_1,\ W_{44}=\boldsymbol{I}_N\otimes(\boldsymbol{R}-2\boldsymbol{M}),$$

$$W_{55}=-(1-\hat{\tau})\boldsymbol{I}_N\otimes\boldsymbol{R}.$$

选择的自适应更新定律如下所示：

$$\dot{k}_i(t)=\alpha_i(\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t)))\boldsymbol{P}_m(\boldsymbol{e}_i(t)-\boldsymbol{D}_m\boldsymbol{e}_i(t-\tau(t))),$$
$$\alpha_i>0,\ i=1,2,\cdots,l. \tag{10}$$

证明 构建如下的 Lyapunov 函数：

$$V(t,\boldsymbol{X})=\sum_{p=1}^{3}V_p(t,\boldsymbol{X}),$$

$$V_1(t,\boldsymbol{X})=\frac{1}{2}\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{X}+\sum_{i=1}^{N}\frac{1}{2\alpha_i}(\boldsymbol{k}_i(t)-\boldsymbol{k}^*)^2,$$

$$V_2(t,\boldsymbol{X})=\int_{t-\tau}^{t}\boldsymbol{e}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Q}_1)\boldsymbol{e}(s)\mathrm{d}s+\int_{t-\tau(t)}^{t}\boldsymbol{e}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Q}_2)\boldsymbol{e}(s)\mathrm{d}s,$$

$$V_3(t,\boldsymbol{X})=\int_{t-\tau(t)}^{t}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(s))(\boldsymbol{I}_N\otimes\boldsymbol{R})f(\boldsymbol{e}(s))\mathrm{d}s.$$

沿着误差系统(7)的轨迹对 $V_1(t,\boldsymbol{X})$ 求无穷小算子：

$$LV_1(t,\boldsymbol{X})=\frac{1}{\alpha}(\boldsymbol{k}(t)-\boldsymbol{k}^*)\dot{\boldsymbol{k}}(t)-\boldsymbol{k}(t)\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{X}+$$
$$\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)[-(\boldsymbol{I}_N\otimes\boldsymbol{C}_m)\boldsymbol{e}(t)+(\boldsymbol{I}_N\otimes\boldsymbol{A}_m)\boldsymbol{f}(\boldsymbol{e}(t))+$$
$$c(\boldsymbol{G}_m\otimes\boldsymbol{\Gamma}_m)\boldsymbol{e}(t)+(\boldsymbol{I}_N\otimes\boldsymbol{B}_m)\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))]+$$
$$\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_\kappa)\boldsymbol{X}+\frac{1}{2}\mathrm{tr}[\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{\sigma}]. \tag{11}$$

由式(8)和假设 2,有

$$\frac{1}{2}\mathrm{trace}(\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{\sigma})\leqslant$$
$$\frac{1}{2}\rho_m h_1\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)+\frac{1}{2}\rho_m h_2\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\boldsymbol{e}(t-\tau(t)). \tag{12}$$

由于

$$\frac{1}{\alpha}(\boldsymbol{k}(t)-\boldsymbol{k}^*)\dot{\boldsymbol{k}}(t)-\boldsymbol{k}(t)\boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{X}=-\boldsymbol{k}^*\boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{X}, \tag{13}$$

$$\boldsymbol{X}^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{X}=\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{P}_m)\boldsymbol{e}(t)-2\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{I}_N\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m)\boldsymbol{e}(t)+$$
$$\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{I}_N\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{D}_m)\boldsymbol{e}(t-\tau(t)), \tag{14}$$

对 $V_2(t,\boldsymbol{X})$ 求无穷小算子,可得

$$LV_2(t,\boldsymbol{X})\leqslant\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(\boldsymbol{Q}_1+\boldsymbol{Q}_2))\boldsymbol{e}(t)-\boldsymbol{e}^{\mathrm{T}}(t-\tau)(\boldsymbol{I}_N\otimes\boldsymbol{Q}_1)\boldsymbol{e}^{\mathrm{T}}(t-\tau)-$$
$$(1-\hat{\tau})\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{I}_N\otimes\boldsymbol{Q}_1)\boldsymbol{e}^{\mathrm{T}}(t-\tau(t)). \tag{15}$$

根据假设 1,有

$$LV_3(t,\boldsymbol{X})\leqslant\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))(\boldsymbol{I}_N\otimes\boldsymbol{R})\boldsymbol{f}(\boldsymbol{e}(t))-$$
$$(1-\hat{\tau})\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t-\tau(t)))(\boldsymbol{I}_N\otimes\boldsymbol{R})\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))+$$
$$2\boldsymbol{e}^{\mathrm{T}}(t)[(\boldsymbol{I}_N\otimes\boldsymbol{LM})\boldsymbol{f}(\boldsymbol{e}(t))-\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))(\boldsymbol{I}_N\otimes\boldsymbol{M})\boldsymbol{f}(\boldsymbol{e}(t))]=$$
$$\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))[\boldsymbol{I}_N\otimes(\boldsymbol{R}-2\boldsymbol{M})]\boldsymbol{f}(\boldsymbol{e}(t))+2\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{LM})\boldsymbol{f}(\boldsymbol{e}(t))-$$

$$(1-\hat{\tau})\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t-\tau(t)))(\boldsymbol{I}_N\otimes\boldsymbol{R})\boldsymbol{f}(\boldsymbol{e}(t-\tau(t))).\tag{16}$$

由式(11)~(16)相加,可得

$$\begin{aligned}
LV(t,\boldsymbol{X}) \leqslant\ & \boldsymbol{e}^{\mathrm{T}}(t)\Bigg[\boldsymbol{I}_N\otimes\Bigg(-\boldsymbol{P}_m\boldsymbol{C}_m+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\frac{1}{2}\rho_m h_1+\\
&\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{P}_\kappa\Bigg)-\boldsymbol{k}^*\otimes\boldsymbol{P}_m+c\boldsymbol{G}_m\otimes\boldsymbol{P}_m\boldsymbol{\Gamma}_m\Bigg]\boldsymbol{e}(t)+\\
&\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\Bigg[\boldsymbol{I}_N\otimes\Bigg(\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{C}_m-2\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_\kappa\Bigg)-c\boldsymbol{G}_m\otimes\boldsymbol{P}_m\boldsymbol{\Gamma}_m\boldsymbol{D}_m\Bigg]\boldsymbol{e}(t)+\\
&\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\Bigg[\boldsymbol{I}_N\otimes\Bigg(\frac{1}{2}\rho h_2-(1-\hat{\tau})\boldsymbol{Q}_2+\\
&\sum_{\kappa=1}^{S}\frac{\gamma_{m\kappa}}{2}\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_\kappa\boldsymbol{D}_m\Bigg)+2\boldsymbol{k}^*\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\Bigg]\boldsymbol{e}(t-\tau(t))-\\
&\boldsymbol{e}^{\mathrm{T}}(t-\tau)(\boldsymbol{I}_N\otimes\boldsymbol{Q}_1)\boldsymbol{e}(t-\tau)+\boldsymbol{e}^{\mathrm{T}}(t)[\boldsymbol{I}_N\otimes(\boldsymbol{P}_m\boldsymbol{A}_m+2\boldsymbol{L}\boldsymbol{M})]\boldsymbol{f}(\boldsymbol{e}(t))+\\
&\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{P}_m\boldsymbol{B}_m)\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))-\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{I}_N\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{A}_m)\boldsymbol{f}(\boldsymbol{e}(t))-\\
&\boldsymbol{e}^{\mathrm{T}}(t-\tau(t))(\boldsymbol{I}_N\otimes\boldsymbol{D}_m^{\mathrm{T}}\boldsymbol{P}_m\boldsymbol{B}_m)\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))+\\
&\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))[\boldsymbol{I}_N\otimes(\boldsymbol{R}-2\boldsymbol{M})]\boldsymbol{f}(\boldsymbol{e}(t))-\\
&(1-\hat{\tau})\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t-\tau(t)))(\boldsymbol{I}_N\otimes\boldsymbol{R})\boldsymbol{f}(\boldsymbol{e}(t-\tau(t)))=\\
&\xi^{\mathrm{T}}(t)\boldsymbol{\Omega}\boldsymbol{\xi}(t),
\end{aligned}\tag{17}$$

其中

$$\boldsymbol{\xi}(t)=[\boldsymbol{e}^{\mathrm{T}}(t)\quad \boldsymbol{e}^{\mathrm{T}}(t-\tau(t))\quad \boldsymbol{e}^{\mathrm{T}}(t-\tau)\quad \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t))\quad \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{e}(t-\tau(t)))]^{\mathrm{T}}.$$

考虑 $\boldsymbol{\Omega}\leqslant\boldsymbol{0}$, 可得

$$LV(t,\boldsymbol{X})\leqslant 0.$$

由式(9)和引理2,有

$$E(LV(t,\boldsymbol{X}))\leqslant-\beta_i\|\boldsymbol{\xi}(t)\|^2\leqslant-\beta\|\boldsymbol{\xi}(t)\|^2\leqslant-\beta\|\boldsymbol{e}(t)\|^2,\tag{18}$$

其中, $-\beta_i=\lambda_{\max}(\Omega)$, $\beta_i>0, i\in S$ 和 $-\beta=\max_{i\in S}\{-\beta_i\}$.由 Dynkin 公式,可以得到

$$\begin{aligned}
&EV(\tau,r(\tau),\boldsymbol{x}(\tau))-EV(0,r(0),\boldsymbol{x}(0))=\\
&\quad E\int_0^\tau LV(s,r(s),\boldsymbol{x}(s))\mathrm{d}s\leqslant-\beta\int_0^\tau\|\boldsymbol{e}(t)\|^2\mathrm{d}s.
\end{aligned}\tag{19}$$

因此由式(19),有

$$\begin{aligned}
&\beta\int_0^\tau\|\boldsymbol{e}(t)\|^2\mathrm{d}s\leqslant\\
&\quad -EV(\tau,r(\tau),\boldsymbol{x}(\tau))+EV(0,r(0),\boldsymbol{x}(0))\leqslant EV(0,r(0),\boldsymbol{x}(0)).
\end{aligned}\tag{20}$$

由式(18)~(20)可得

$$E\int_0^\tau\|\boldsymbol{e}(t)\|^2\mathrm{d}t\leqslant\frac{1}{\beta}EV(0,r(0),\boldsymbol{x}(0))<\infty.$$

根据引理4,系统(1)和系统(3)是同步的,证毕.

3 数值仿真

在这部分,通过给出实例来证明结果的有效性.Markov 链进程 $\{r(t),t\geqslant 0\}$ 在 $S=\{1,2\}$ 中取值.给出概率转移矩阵,产生一个 Markov 链.考虑给出的概率转移矩阵如下:

$$\boldsymbol{\Pi}=\begin{bmatrix}-1.2 & 1.2\\ 0.5 & -0.5\end{bmatrix}.$$

考虑具有四个节点的中立型神经网络(3)：

$$\begin{aligned}&\mathrm{d}[\boldsymbol{x}_i(t)-\boldsymbol{D}_m\boldsymbol{x}_i(t-\tau(t))]=\\&\quad\Big[-\boldsymbol{C}_m\boldsymbol{x}_i(t)+\boldsymbol{A}_m\boldsymbol{f}(\boldsymbol{x}_i(t))+\boldsymbol{B}_m\boldsymbol{f}(\boldsymbol{x}_i(t-\tau(t)))+\boldsymbol{J}+\\&\quad+c\sum_{j=1}^{4}g_{ij}^{m}\boldsymbol{\Gamma}_m\boldsymbol{x}_j(t)+\boldsymbol{R}_i(t)\Big]\mathrm{d}t+\\&\quad\boldsymbol{\sigma}(r(t),\boldsymbol{x}_i(t),\boldsymbol{x}_i(t-\tau(t)))\mathrm{d}\boldsymbol{\omega}(t),\qquad i=1,2,3,4,\end{aligned}\tag{21}$$

其中，$\boldsymbol{x}_i(t)=[x_{i1}(t),x_{i2}(t)]^{\mathrm{T}}$ 是第 i 节点的状态变量，

$$\boldsymbol{f}(\boldsymbol{x}(t))=\tanh(\boldsymbol{x}),\ \tau(t)=\frac{\mathrm{e}^t}{1+\mathrm{e}^t},\ c=2,$$

同时其他参数如下所示：

$$\boldsymbol{D}_1=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.2\end{bmatrix},\ \boldsymbol{C}_1=\begin{bmatrix}-1 & 0\\ 0 & -1.1\end{bmatrix},\ \boldsymbol{A}_1=\begin{bmatrix}1.9 & -0.1\\ -4.7 & 4.3\end{bmatrix},$$

$$\boldsymbol{B}_1=\begin{bmatrix}-1.3 & -0.1\\ -0.7 & -3.7\end{bmatrix},\ \boldsymbol{G}_1=\begin{bmatrix}-3 & 1 & 1 & 1\\ 1 & -3 & 1 & 1\\ 1 & 1 & -3 & 1\\ 1 & 1 & 1 & -3\end{bmatrix},\ \boldsymbol{\Gamma}_1=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix},$$

$$\boldsymbol{D}_2=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.1\end{bmatrix},\ \boldsymbol{C}_2=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},\ \boldsymbol{A}_2=\begin{bmatrix}2 & -0.1\\ -5 & 4.5\end{bmatrix},$$

$$\boldsymbol{B}_2=\begin{bmatrix}-1.5 & -0.1\\ -0.2 & -4\end{bmatrix},\ \boldsymbol{G}_2=\begin{bmatrix}-3.1 & 0.4 & 1.5 & 0.6\\ 1 & -3 & 1 & 0.8\\ 0.9 & 0.7 & -2.7 & 1\\ 1.1 & 0.9 & 1 & -3\end{bmatrix},\ \boldsymbol{\Gamma}_2=\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix}.$$

噪声强度矩阵为

$$\boldsymbol{\sigma}_1(t,\boldsymbol{x}(t),\boldsymbol{x}(t-\tau(t)))=\begin{bmatrix}0.5\boldsymbol{x}(t)\\ 0.4\boldsymbol{x}(t-\tau(t))\end{bmatrix},$$

$$\boldsymbol{\sigma}_2(t,\boldsymbol{x}(t),\boldsymbol{x}(t-\tau(t)))=\begin{bmatrix}0.6\boldsymbol{x}(t)\\ 0.5\boldsymbol{x}(t-\tau(t))\end{bmatrix}.$$

噪声强度矩阵 $\boldsymbol{\sigma}(\cdot,\cdot,\cdot)$ 满足有界条件，且 $h_1=h_2=0.45$。当 $r(t)=1$ 时，将控制器添加到第一个节点，$\boldsymbol{k}^*=\mathrm{diag}\{30,0,0,0\}$；当 $r(t)=2$ 时，将控制器添加到前三个节点中，$\boldsymbol{k}^*=\mathrm{diag}\{30,30,30,0\}$。使用 LMI 工具箱，获得下列的可行解：

$$\rho_1=202.8679,\ \rho_2=135.5996,$$

$$\boldsymbol{R}=\begin{bmatrix}111.4742 & 15.0013\\ 15.0013 & 162.5376\end{bmatrix},$$

$$\boldsymbol{P}_1=\begin{bmatrix}47.3495 & -4.7489\\ -4.7489 & 36.2643\end{bmatrix},\ \boldsymbol{P}_2=\begin{bmatrix}48.1152 & -2.9823\\ -2.9823 & 33.3877\end{bmatrix},$$

$$\boldsymbol{Q}_1=\begin{bmatrix}622.2115 & 6.2484\\ 6.2484 & 229.8541\end{bmatrix},\ \boldsymbol{Q}_2=\begin{bmatrix}77.9193 & -48.3705\\ -48.3705 & 156.3748\end{bmatrix}.$$

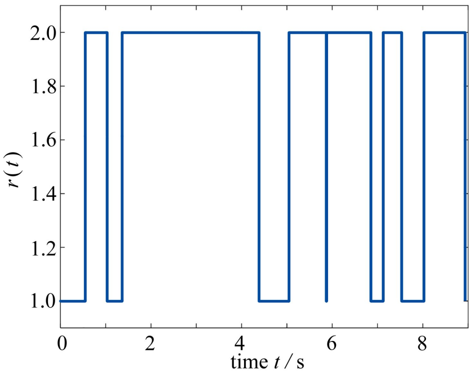

图 1 Markov 链的轨迹变化

Fig. 1 Markovian chain trajectory changes

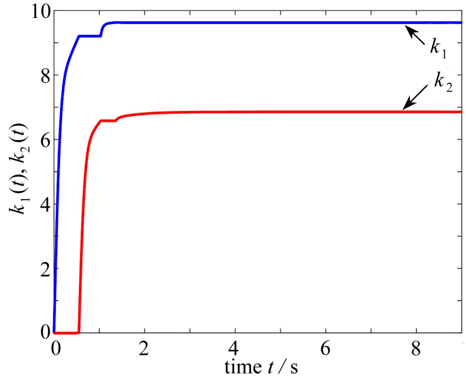

图 2 控制器增益的时间演变

Fig. 2 Time evolution of the adaptive strengths

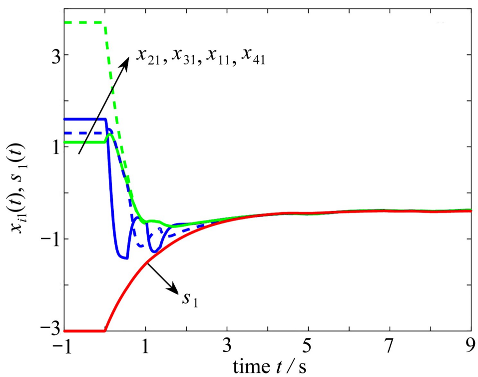

图 3 控制网络(21) $x_{i1}(t)$ $(i = 1,2,3,4)$ 轨迹变化

Fig. 3 The trajectories of the state variables of $x_{i1}(t)$ $(i = 1,2,3,4)$ in controlled network (21)

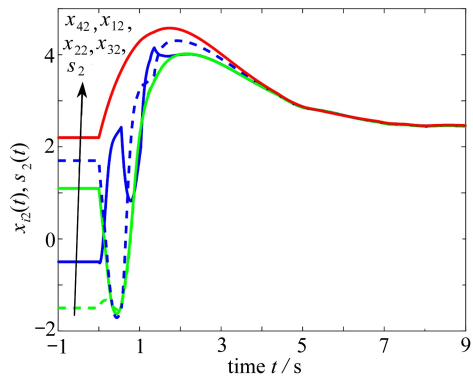

图 4 控制网络(21) $x_{i2}(t)$ $(i = 1,2,3,4)$ 轨迹变化

Fig. 4 The trajectories of the state variables of $x_{i2}(t)$ $(i = 1,2,3,4)$ in controlled network (21)

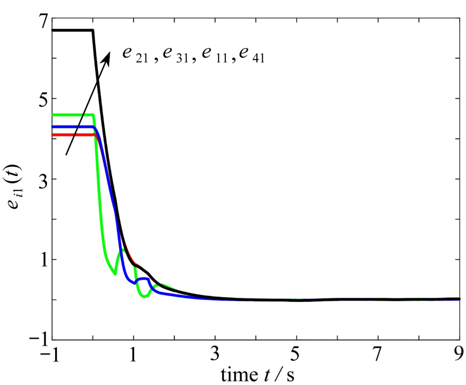

图 5 误差 $e_{i1}(t)$ 轨迹变化

Fig. 5 The trajectories of the state variables of $e_{i1}(t)$

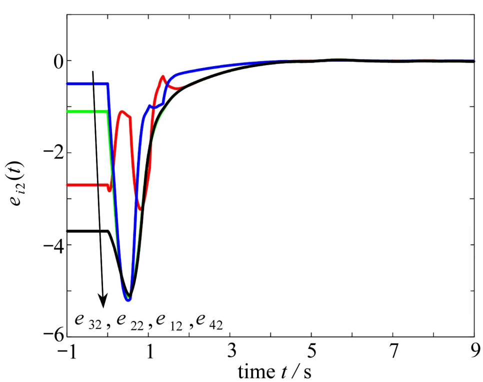

图 6 误差 $e_{i2}(t)$ 状态变化

Fig. 6 The trajectories of the state variables of $e_{i2}(t)$

表 1 给出了最大时滞上界.图 1~6 所展示的仿真结果给定的系统初始值条件为 $\boldsymbol{x}_1 = [1.6. - 0.5]$, $\boldsymbol{x}_2 = [1.1, 1.1]$, $\boldsymbol{x}_3 = [1.3, 1.7]$, $\boldsymbol{x}_4 = [3.7, -1.5]$, $\boldsymbol{s} = [-3.0, 2.2]$ 以及 $\alpha_1 = \alpha_2 = 1$.图 1 为 Markov 链的变化曲线;图 2 是控制器增益的变化曲线,可以看到控制增益在几秒内达到稳定;图 3、4 表示的是 $\boldsymbol{x}_i(t)$ $(i = 1,2,3,4)$ 和 $s_i(t)$ 的状态变化,可以看出式(21)和式(1)

达到同步;图 5、6 是误差的变化曲线,随着时间的流逝,状态演化误差迅速接近零.

表 1 最大时滞上界

Table 1 The maximum delay bounds

method	ref. [1]	ref. [2]	ref. [26]	theorem 1
$\max(\tau(t))$	0.549 3	0.549 6	0.671 0	1

4 结 论

本文研究了具有 Markov 和随机扰动的中立型耦合神经网络的自适应同步问题.将自适应牵制控制运用到中立型耦合神经网络中,即通过控制网络中的部分节点,以达到控制整个网络的目的,降低了控制的成本.通过构造合适的 Lyapunov 函数,得到了误差中立型耦合神经网络的稳定性判据,数值仿真算例验证了理论分析的有效性.在未来的研究工作中:1) 我们将研究更简单的控制器,确保具有时变时滞和 Markov 切换的中立型系统的同步;2) 将进一步研究带有混合时滞的中立型系统的同步问题.

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Adaptive Synchronization of Neutral-Type Coupled Neural Networks With Stochastic Perturbations and Markovian Jumpings

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Abstract: The adaptive synchronization problem of neutral-type neural networks with time-varying delays and stochastic perturbations was discussed. Stochastic perturbations were described as the Brownian motion. Through the Lyapunov stability theory, the LMI analysis techniques and the matrix theory were used to study the adaptive synchronization of neutral-type neural networks with stochastic perturbations and Markovian jumpings. The sufficient conditions for the system synchronization were given and proved. The criterion for adaptive synchronization of neutral-type neural networks with time-varying delays and stochastic perturbations was obtained. Finally, numerical examples show the effectiveness and applicability of the proposed approach.

Key words: neutral-type neural network; stochastic perturbation; pinning control; adaptive synchronization