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大尺度湿大气原始方程组对边界参数的连续依赖性*

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摘要： 大气的大尺度动力学方程由 Navier-Stokes 方程导出的原始方程组控制,并与热力学和盐度扩散输运方程耦合.在过去的几十年里,人们从数学的角度对大气、海洋与耦合了大气和海洋的原始方程组进行了广泛的研究.许多学者的研究主要关注原始方程组在数学上的逻辑性,即方程组的适定性.笔者开始注意到研究原始方程组自身稳定性的必要性.因为在模型建立、简化的过程中不可避免地会出现一些误差,这就需要研究方程组中系数的微小变化是否会引起方程组解的巨大变化.该文运用原始方程组解的先验估计,结合能量估计与微分不等式技术,展示了如何控制水汽比,证明了大尺度湿大气原始方程组的解对边界参数的连续依赖性.

关　键　词： 原始方程组； 先验估计； 连续依赖性

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引　　言

大气的大尺度动力学方程由 Navier-Stokes 方程导出的原始方程组控制,并与热力学和盐度扩散输运方程耦合.在过去的几十年里,人们从数学的角度对大气、海洋与耦合了大气和海洋的原始方程组进行了广泛的研究[1-3].在文献[4]中,Lions、Temam 和 Wang 引入了大尺度海洋的原始方程,证明了大尺度海洋原始方程弱解的存在性和局部时间强解的适定性.基于 Lions、Temam 和 Wang 的研究[5],许多学者继续考虑了大规模大气原始方程解的适定性[6-8].关于大气、海洋三维黏性原始方程组强解的整体存在性、唯一性以及对于初值的连续依赖性,可见文献[9-14].文献[15-16]研究了在气压坐标下,湿大气原始方程弱解的整体存在性.文献[17]解决了湿大气原始方程整体适定性问题,以及解的长时间行为.此外,人们还研究了带其他边界条件的大气、海洋原始方程组的整体适定性[18-20].

显然,上述研究主要关注原始方程组在数学上的逻辑性,即方程组的适定性.我们开始注意到研究原始方程组自身稳定性的必要性.因为在模型建立、简化的过程中不可避免地会出现

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一些误差,这就需要研究方程组中系数的微小变化是否会引起方程组解的巨大变化.在文献中已经出现了相关研究[19-20],进而得到了三个类似结果[21-25].

本文将继续这方面的研究工作,但是我们在方程组中考虑了水汽比的影响,这种类型的方程组称为湿大气原始方程组.本文将研究该方程组对边界参数的连续依赖性.

本文研究在 $\Omega\times\infty$ 上,如下三维大尺度湿大气原始方程组[6,26]:

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}+w\frac{\partial \boldsymbol{u}}{\partial z}+\nabla\Phi+\frac{1}{R_{\mathrm{o}}}f\boldsymbol{u}^{\perp}+\mathrm{L}_1\boldsymbol{u}=\boldsymbol{0},\tag{1}$$

$$\frac{\partial \Phi}{\partial z}+\frac{bP}{p}(1+aq)T=0,\tag{2}$$

$$\nabla\cdot\boldsymbol{u}+\frac{\partial w}{\partial z}=0,\tag{3}$$

$$\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T+w\frac{\partial T}{\partial z}-\frac{bP}{p}(1+aq)w+\mathrm{L}_2T=Q_1,\tag{4}$$

$$\frac{\partial q}{\partial t}+\boldsymbol{u}\cdot\nabla q+w\frac{\partial q}{\partial z}+\mathrm{L}_3q=Q_2,\tag{5}$$

其中 Ω 为 R^3 上的一个柱形区域,

$$\Omega=M\times(0,1),\tag{6}$$

M 是 R^2 上的一个光滑有界区域, Ω 的边界记为

$$\Gamma_{\mathrm{u}}=\{(x,y,z)\in\bar{\Omega}:\ z=1\},$$

$$\Gamma_{\mathrm{b}}=\{(x,y,z)\in\bar{\Omega}:\ z=0\},$$

$$\Gamma_{1}=\{(x,y,z)\in\bar{\Omega}:\ (x,y)\in\partial M,\ 0\leqslant z\leqslant 1\}.$$

$\boldsymbol{u}=(u_1,u_2)$ 表示水平速度场, $\boldsymbol{u}^{\perp}=(-u_2,u_1)$,三维速度场$(u_1,u_2,w)$、温度 T 均为未知函数. R_{o} 表示 Rossby 系数, $f=2\cos\theta_0$ 表示 Coriolis 参数,未知函数 q 表示空气中水汽混合比, Φ 是位势场,压力 P 满足 $p=(P-p_0)z+p_0(0<p_0\leqslant p\leqslant P)$, Q_1,Q_2 为分别给定的热源与水汽源函数. $\nabla=(\partial_x,\partial_y)$ 是水平梯度算子, a,b 均为大于零的常数, $a\approx 0.618$.黏性和扩散算子 $\mathrm{L}_1,\mathrm{L}_2,\mathrm{L}_3$ 分别为

$$\mathrm{L}_1=-\mu_1\Delta-\mu_2\frac{\partial^2}{\partial z^2},$$

$$\mathrm{L}_2=-\mu_3\Delta-\mu_4\frac{\partial^2}{\partial z^2},$$

$$\mathrm{L}_3=-\mu_5\Delta-\mu_6\frac{\partial^2}{\partial z^2},$$

其中 μ_1,μ_2 是大于零的常数,分别表示水平和垂直方向的 Reynolds 系数, μ_3,μ_4,μ_5,μ_6 均为大于零的常数, $\Delta=\partial_x^2+\partial_y^2$ 是 Laplace 算子.原始方程组(1)~(5)满足的边界条件为

$$\left.\frac{\partial \boldsymbol{u}}{\partial z}\right|_{\Gamma_{\mathrm{u}}}=\boldsymbol{0},\ w|_{\Gamma_{\mathrm{u}}}=0,\ \left.\left(\mu_4\frac{\partial T}{\partial z}+\alpha T\right)\right|_{\Gamma_{\mathrm{u}}}=0,\ \left.\left(\mu_6\frac{\partial q}{\partial z}+\beta q\right)\right|_{\Gamma_{\mathrm{u}}}=0,\tag{7}$$

$$\left.\frac{\partial \boldsymbol{u}}{\partial z}\right|_{\Gamma_{\mathrm{b}}}=\boldsymbol{0},\ w|_{\Gamma_{\mathrm{b}}}=0,\ \left.\frac{\partial T}{\partial z}\right|_{\Gamma_{\mathrm{b}}}=0,\ \left.\frac{\partial q}{\partial z}\right|_{\Gamma_{\mathrm{b}}}=0,\tag{8}$$

$$\boldsymbol{u}\cdot\boldsymbol{n}\mid_{\Gamma_1}=0,\ \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}\times\boldsymbol{n}\Big|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial T}{\partial \boldsymbol{n}}\Big|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial q}{\partial \boldsymbol{n}}\Big|_{\Gamma_1}=\boldsymbol{0}, \tag{9}$$

其中 $\boldsymbol{n}$ 是 Γ_1 上的单位外法向量, α,β 是大于零的常数.初始条件满足

$$\boldsymbol{u}(x,y,z,0)=\boldsymbol{u}_0(x,y,z),\ T(x,y,z,0)=T_0(x,y,z),\ q(x,y,z,0)=q_0(x,y,z), \tag{10}$$

其中 $\boldsymbol{u}_0,T_0,q_0$ 是可微函数.

为简便运算,不妨假设 $\mu_1,\mu_2,\mu_3,\mu_4,\mu_5,\mu_6$ 等于 1,并且记算子 $\mathrm{L}=-\Delta-\partial^2/\partial z^2$.将式(3)在 $(0,z)$ 上积分,得

$$w(x,y,z,t)=w(x,y,0,t)-\int_0^z\nabla\cdot\boldsymbol{u}(x,y,s,t)\,\mathrm{d}s.$$

由边界条件(7)和(8),可得

$$w(x,y,z,t)=-\int_0^z\nabla\cdot\boldsymbol{u}(x,y,s,t)\,\mathrm{d}s, \tag{11}$$

以及

$$\int_0^1\nabla\cdot\boldsymbol{u}(x,y,s,t)\,\mathrm{d}s=\nabla\cdot\int_0^1\boldsymbol{u}(x,y,s,t)\,\mathrm{d}s=0. \tag{12}$$

假设 Φ_s 为等压面 $s=1$ 上的未知函数.将式(2)在 $(0,z)$ 上积分,得到

$$\Phi(x,y,z,t)=\Phi_s-\int_0^z\frac{bP}{p(s)}(1+aq(x,y,s,t))T(x,y,s,t)\,\mathrm{d}s. \tag{13}$$

由式(11)~(13),可以将原方程组(1)~(5)改写为

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}-\left(\int_0^z\nabla\cdot\boldsymbol{u}\,\mathrm{d}s\right)\frac{\partial \boldsymbol{u}}{\partial z}-\int_0^z\frac{bP}{p}\nabla(1+aq)T\,\mathrm{d}s+$$
$$\nabla\Phi_s+\frac{1}{R_{\mathrm{o}}}f\boldsymbol{u}^{\perp}+\mathrm{L}\boldsymbol{u}=0, \tag{14}$$

$$\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T-\left(\int_0^z\nabla\cdot\boldsymbol{u}\,\mathrm{d}s\right)\frac{\partial T}{\partial z}+\frac{bP}{p}(1+aq)\left(\int_0^z\nabla\cdot\boldsymbol{u}\,\mathrm{d}s\right)+\mathrm{L}T=Q_1, \tag{15}$$

$$\frac{\partial q}{\partial t}+\boldsymbol{u}\cdot\nabla q-\left(\int_0^z\nabla\cdot\boldsymbol{u}\,\mathrm{d}s\right)\frac{\partial q}{\partial z}+\mathrm{L}q=Q_2. \tag{16}$$

系统(14)~(16)的边界条件为

$$\frac{\partial \boldsymbol{u}}{\partial z}\Big|_{\Gamma_{\mathrm{u}}}=\boldsymbol{0},\ \frac{\partial T}{\partial z}\Big|_{\Gamma_{\mathrm{u}}}=-\alpha T,\ \frac{\partial q}{\partial z}\Big|_{\Gamma_{\mathrm{u}}}=-\beta q, \tag{17}$$

$$\frac{\partial \boldsymbol{u}}{\partial z}\Big|_{\Gamma_{\mathrm{b}}}=\boldsymbol{0},\ \frac{\partial T}{\partial z}\Big|_{\Gamma_{\mathrm{b}}}=0,\ \frac{\partial q}{\partial z}\Big|_{\Gamma_{\mathrm{b}}}=0, \tag{18}$$

$$\boldsymbol{u}\cdot\boldsymbol{n}\mid_{\Gamma_1}=0,\ \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}\times\boldsymbol{n}\Big|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial T}{\partial \boldsymbol{n}}\Big|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial q}{\partial \boldsymbol{n}}\Big|_{\Gamma_1}=\boldsymbol{0}. \tag{19}$$

初始条件为

$$\boldsymbol{u}(x,y,z,0)=\boldsymbol{u}_0(x,y,z),\ T(x,y,z,0)=T_0(x,y,z),\ q(x,y,z,0)=q_0(x,y,z). \tag{20}$$

1 先 验 估 计

采用文献[16]的方法(见文献[16]中式(3.9)、(3.32)、(3.37)、(3.44)),类似可得如下引理 1.

引理 1[16]　设 $Q_1 \in L^2(\Omega)$，$Q_2 \in L^2(\Omega)$，$(\boldsymbol{v},T,q)$ 是方程组(14)~(20)的解，则

1) $\|\boldsymbol{v}\|_2^2 + \|T\|_2^2 + \int_0^t (\|\boldsymbol{v}(\tau)\|^2 + \|T(\tau)\|^2)\mathrm{d}\tau \leqslant \rho_2(t)$,

2) $\|\partial_z \boldsymbol{v}\|_2^2 + \int_0^t (\|\partial_z \boldsymbol{v}(\tau)\|^2 + \|T(\tau)\|^2)\mathrm{d}\tau \leqslant \rho_8(t)$,

3) $\beta\|q\|_{L^2(\Gamma_u)}^2 + \alpha\|T\|_{L^2(\Gamma_u)}^2 + \int_0^t \|\nabla q_z\|_2^2\mathrm{d}\tau + \int_0^t \|\nabla T_z\|_2^2\mathrm{d}\tau \leqslant \rho_9(t)$,

4) $\|\nabla \boldsymbol{v}\|_2^2 + \|\nabla q\|_2^2 + \|\nabla T\|_2^2 + \int_0^t \|\nabla\partial_z q\|_2^2 +$

$$\int_0^t \|\nabla\partial_z T\|_2^2\mathrm{d}\tau + \int_0^t \|\nabla\partial_z \boldsymbol{v}\|_2^2\mathrm{d}\tau \leqslant \rho_{10}(t),$$

其中 $\rho_2(t)$，$\rho_8(t)$，$\rho_9(t)$，$\rho_{10}(t)$ 是关于 t 的正函数.

引理 2[27]　设 $\Omega_1 \subset R^{m_1}$，且 $\Omega_2 \subset R^{m_2}$，其中 m_1 和 m_2 是正整数，函数 $f(\xi,\eta)$ 是 $\Omega_1 \times \Omega_2$ 上的可测函数，则

$$\left[\int_{\Omega_1}\left(\int_{\Omega_2} |f(\xi,\eta)|\,\mathrm{d}\eta\right)^p \mathrm{d}\xi\right]^{1/p} \leqslant \int_{\Omega_2}\left(\int_{\Omega_1} |f(\xi,\eta)|^p \mathrm{d}\xi\right)^{1/p} \mathrm{d}\eta \cdot p \geqslant 1. \tag{21}$$

引理 3[28]　设 Ω 是有界的凸区域，则

$$\|\boldsymbol{u}\|_{L^4(\Omega)}^2 \leqslant k\left(1 + \frac{\delta}{4}\right)\|\boldsymbol{u}\|_{L^2(\Omega)}^2 + \frac{3}{4}\delta^{-1/3}\|\nabla \boldsymbol{u}\|_{L^2(\Omega)}^2, \qquad k > 0, \tag{22}$$

其中 δ 是大于零的任意常数.

2　方程组对边界参数 α，β 的连续依赖性

假设 $(\boldsymbol{u}_1,T_1,q_1,\boldsymbol{\Phi}_{s_1})$ 和 $(\boldsymbol{u}_2,T_2,q_2,\boldsymbol{\Phi}_{s_2})$ 是方程组(14)~(20)对应于不同边界参数 α_1，β_1 和 α_2，β_2 的两组解，记

$$\begin{cases} \tilde{\boldsymbol{u}} = \boldsymbol{u}_1 - \boldsymbol{u}_2,\ \tilde{T} = T_1 - T_2,\ \tilde{q} = q_1 - q_2, \\ \tilde{\boldsymbol{\Phi}}_s = \boldsymbol{\Phi}_{s_1} - \boldsymbol{\Phi}_{s_2},\ \tilde{\alpha} = \alpha_1 - \alpha_2,\ \tilde{\beta} = \beta_1 - \beta_2. \end{cases} \tag{23}$$

则 $\tilde{\boldsymbol{u}}$，$\tilde{T}$，$\tilde{q}$，$\tilde{\boldsymbol{\Phi}}_s$ 满足

$$\frac{\partial \tilde{\boldsymbol{u}}}{\partial t} + (\boldsymbol{u}_1\cdot\nabla)\tilde{\boldsymbol{u}} - \left(\int_0^z \nabla\cdot\boldsymbol{u}_1\mathrm{d}s\right)\frac{\partial \tilde{\boldsymbol{u}}}{\partial z} - \int_0^z \frac{bP}{p(s)}\nabla[(1 + aq_1)\tilde{T}]\mathrm{d}s + (\tilde{\boldsymbol{u}}\cdot\nabla)\boldsymbol{u}_2 -$$

$$\left(\int_0^z \nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial \boldsymbol{u}_2}{\partial z} - \int_0^z \frac{abP}{p(s)}\nabla\tilde{q}\cdot T_2\mathrm{d}s + \nabla\tilde{\boldsymbol{\Phi}}_s + \frac{1}{R_o}f\tilde{\boldsymbol{u}}^{\perp} + \mathrm{L}\tilde{\boldsymbol{u}} = 0, \tag{24}$$

$$\frac{\partial \tilde{T}}{\partial t} + \boldsymbol{u}_1\cdot\nabla\tilde{T} - \left(\int_0^z \nabla\cdot\boldsymbol{u}_1\mathrm{d}s\right)\frac{\partial \tilde{T}}{\partial z} + \frac{bP}{p}(1 + aq_1)\left(\int_0^z \nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right) + \tilde{\boldsymbol{u}}\cdot\nabla T_2 -$$

$$\left(\int_0^z \nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial T_2}{\partial z} + \frac{abP}{p(s)}\tilde{q}\int_0^z \nabla\cdot\boldsymbol{u}_2\mathrm{d}s + \mathrm{L}\tilde{T} = 0, \tag{25}$$

$$\frac{\partial \tilde{q}}{\partial t} + \boldsymbol{u}_1\cdot\nabla\tilde{q} + \tilde{\boldsymbol{u}}\cdot\nabla q_2 - \left(\int_0^z \nabla\cdot\boldsymbol{u}_1\mathrm{d}s\right)\frac{\partial \tilde{q}}{\partial z} - \left(\int_0^z \nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial q_2}{\partial z} + \mathrm{L}\tilde{q} = 0. \tag{26}$$

边界条件为

$$\left.\frac{\partial \tilde{\boldsymbol{u}}}{\partial z}\right|_{\Gamma_u} = \boldsymbol{0},\ \left.\frac{\partial \tilde{T}}{\partial z}\right|_{\Gamma_u} = -\alpha_1\tilde{T} - \tilde{\alpha}T_2,\ \left.\frac{\partial \tilde{q}}{\partial z}\right|_{\Gamma_u} = -\beta_1\tilde{q} - \tilde{\beta}q_2, \tag{27}$$

$$\frac{\partial\tilde{\boldsymbol{u}}}{\partial z}\bigg|_{\Gamma_b}=\boldsymbol{0},\ \frac{\partial\tilde{T}}{\partial z}\bigg|_{\Gamma_b}=0,\ \frac{\partial\tilde{q}}{\partial z}\bigg|_{\Gamma_b}=0,\tag{28}$$

$$\tilde{\boldsymbol{u}}\cdot\boldsymbol{n}\mid_{\Gamma_1}=0,\ \frac{\partial\tilde{\boldsymbol{u}}}{\partial\boldsymbol{n}}\times\boldsymbol{n}\bigg|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial\tilde{T}}{\partial\boldsymbol{n}}\bigg|_{\Gamma_1}=\boldsymbol{0},\ \frac{\partial\tilde{q}}{\partial\boldsymbol{n}}\bigg|_{\Gamma_1}=\boldsymbol{0}.\tag{29}$$

初始条件为

$$\tilde{\boldsymbol{u}}(x,y,z,0)=\boldsymbol{0},\ \tilde{T}(x,y,z,0)=0,\ \tilde{q}(x,y,z,0)=0.\tag{30}$$

定理 1 假设 $(\boldsymbol{u}_1,T_1,q_1)$ 和 $(\boldsymbol{u}_2,T_2,q_2)$ 是方程组(14)~(20)对应于不同边界参数 α_1，β_1 和 α_2,β_2 的两组解，则对任意 $t>0$，当 $\alpha_1\to\alpha_2$,$\beta_1\to\beta_2$ 时，有

$$(\boldsymbol{u}_1,T_1,q_1)\to(\boldsymbol{u}_2,T_2,q_2),$$

且满足如下不等式：

$$\|\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\|\tilde{T}\|^2_{L^2(\Omega)}+\|\tilde{q}\|^2_{L^2(\Omega)}\leqslant$$
$$\mathrm{e}^{\int_0^t m(s)\mathrm{d}s}\left[\frac{\tilde{\alpha}^2}{2\alpha_1}\int_0^t\|T_2(s)\|^2_{L^2(M)}\mathrm{d}s+\frac{\tilde{\beta}^2}{2\beta_1}\int_0^t\|q_2(s)\|^2_{L^2(M)}\mathrm{d}s\right].$$

也即表明方程组(14)~(20)的解对边界参数的连续依赖性.

证明 将方程(24)和(25)分别与 $\tilde{\boldsymbol{u}}$ 和 $\tilde{T}$ 在 $L^2(\Omega)$ 中做内积，应用分部积分，由边界条件(27)~(29)，计算得

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\|\partial_z\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}=$$
$$-\int_\Omega\left[(\tilde{\boldsymbol{u}}\cdot\nabla)\boldsymbol{u}_2-\int_0^z\frac{bP}{p(s)}\nabla[(1+aq_1)\tilde{T}]\mathrm{d}s\right]\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z-$$
$$\int_\Omega\left[\left(\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial\boldsymbol{u}_2}{\partial z}-\int_0^z\frac{abP}{p(s)}\nabla(\tilde{q}T_2)\mathrm{d}s\right]\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z,\tag{31}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\tilde{T}\|^2_{L^2(\Omega)}+\|\nabla\tilde{T}\|^2_{L^2(\Omega)}+\|\partial_z\tilde{T}\|^2_{L^2(\Omega)}=$$
$$\int_\Omega\left[(\tilde{\boldsymbol{u}}\cdot\nabla)T_2+\frac{bP}{p(s)}\left[(1+aq_1)\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right]\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z-\right.$$
$$\left.\int_\Omega\left(\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial T_2}{\partial z}+\frac{abP}{p(s)}\tilde{q}\int_0^z\nabla\cdot\boldsymbol{u}_2\mathrm{d}s\right]\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z-$$
$$\alpha_1\int_M\tilde{T}^2\mathrm{d}x\mathrm{d}y-\tilde{\alpha}\int_M T_2\tilde{T}\mathrm{d}x\mathrm{d}y.\tag{32}$$

我们用到了以下计算结果：

$$\int_\Omega\left[(\boldsymbol{u}_1\cdot\nabla)\tilde{\boldsymbol{u}}-\left(\int_0^z\nabla\cdot\boldsymbol{u}_1\mathrm{d}s\right)\frac{\partial\tilde{\boldsymbol{u}}}{\partial z}\right]\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z=0,\tag{33}$$

$$\int_\Omega\left[(\boldsymbol{u}_1\cdot\nabla)\tilde{T}-\left(\int_0^z\nabla\cdot\boldsymbol{u}_1\mathrm{d}s\right)\frac{\partial\tilde{T}}{\partial z}\right]\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z=0,\tag{34}$$

$$\int_\Omega\left(\nabla\tilde{\Phi}_s+\frac{1}{R_o}f\tilde{\boldsymbol{u}}^\perp\right)\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z=0.\tag{35}$$

由 Hölder、Cauchy、Minkowsky 不等式与引理 1，计算得

$$-\int_\Omega(\tilde{\boldsymbol{u}}\cdot\nabla)\boldsymbol{u}_2\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z\leqslant$$

$$
\begin{aligned}
&\|\tilde{\boldsymbol{u}}\|^2_{L^4(\Omega)}\|\nabla \boldsymbol{u}_2\|_{L^2(\Omega)} \leqslant \\
&\left[k\left(1+\frac{\delta}{4}\right)\|\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{3}{4}\delta^{-1/3}\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}\right]\sqrt{\rho_{10}(t)} \leqslant \\
&k\left(1+\frac{\delta}{4}\right)\sqrt{\rho_{10}(t)}\|\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{3}{4}\delta^{-1/3}\sqrt{\rho_{10}(t)}\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)},
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
&-\int_\Omega\left(\int_0^z \nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial \boldsymbol{u}_2}{\partial z}\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z \leqslant \\
&\left[\int_M\left(\int_0^1|\tilde{\boldsymbol{u}}|^2\mathrm{d}z\right)^2\mathrm{d}x\mathrm{d}y\right]^{1/4}\left[\int_M\left(\int_0^1|\partial_z\boldsymbol{u}_2|^2\mathrm{d}z\right)^2\mathrm{d}x\mathrm{d}y\right]^{1/4}\times \\
&\left[\int_M\left(\int_0^1|\nabla\tilde{\boldsymbol{u}}|\mathrm{d}z\right)^2\mathrm{d}x\mathrm{d}y\right]^{1/2} \leqslant \\
&\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}\left(\int_0^1\|\tilde{\boldsymbol{u}}\|^2_{L^4(M)}\mathrm{d}z\right)^{1/2}\left(\int_0^1\|\partial_z\boldsymbol{u}_2\|^2_{L^4(M)}\mathrm{d}z\right)^{1/2} \leqslant \\
&\epsilon_1\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{C_0^2}{4\epsilon_1}\|\partial_z\boldsymbol{u}_2\|_{L^2(\Omega)}\|\nabla\partial_z\boldsymbol{u}_2\|_{L^2(\Omega)}\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)} \leqslant \\
&\left(\epsilon_1+\frac{C_0^2\epsilon_2}{8\epsilon_1}\right)\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{C_0^2\rho_8(t)\rho_{10}(t)}{8\epsilon_1\epsilon_2}\|\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}.
\end{aligned} \tag{37}
$$

与式(36)和(37)的计算类似,可得

$$
\begin{aligned}
&-\int_\Omega\int_0^z\frac{abP}{p(s)}\nabla(\tilde{q}T_2)\mathrm{d}s\cdot\tilde{\boldsymbol{u}} = \\
&\int_\Omega\int_0^z\frac{abP}{p(s)}\tilde{q}T_2\mathrm{d}s\,\nabla\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z \leqslant \\
&\frac{abP}{p_0}\int_M\left(\int_0^1|\tilde{q}||T_2|\mathrm{d}z\right)\left(\int_0^1|\nabla\tilde{\boldsymbol{u}}|\mathrm{d}z\right)\mathrm{d}x\mathrm{d}y \leqslant \\
&C\int_M\left[\left(\int_0^1|\tilde{q}|^2\mathrm{d}z\right)^{1/2}\left(\int_0^1|\tilde{T}_2|^2\mathrm{d}z\right)^{1/2}\left(\int_0^1|\nabla\tilde{\boldsymbol{u}}|\mathrm{d}z\right)\right]\mathrm{d}x\mathrm{d}y \leqslant \\
&C\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}\left(\int_0^1\|\tilde{q}\|^2_{L^4(M)}\mathrm{d}z\right)^{1/2}\left(\int_0^1\|\tilde{T}_2\|^2_{L^4(M)}\mathrm{d}z\right)^{1/2} \leqslant \\
&C\epsilon_3\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{C_0^2}{8\epsilon_3\epsilon_4}\|\tilde{q}\|^2_{L^2(\Omega)}+\frac{C_0^2\epsilon_4}{8\epsilon_3}\rho_3(t)\rho_{10}(t)\|\nabla\tilde{q}\|^2_{L^2(\Omega)}.
\end{aligned} \tag{38}
$$

利用 Hölder、Cauchy、Minkowsky 不等式与引理 1,类似式(36)~(38),计算得

$$
\begin{aligned}
&-\int_\Omega(\tilde{\boldsymbol{u}}\cdot\nabla)T_2\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z \leqslant \\
&\left(\int_\Omega|\tilde{\boldsymbol{u}}|^4\mathrm{d}x\mathrm{d}y\mathrm{d}z\right)^{1/4}\left(\int_\Omega|\nabla T_2|^2\mathrm{d}x\mathrm{d}y\mathrm{d}z\right)^{1/2}\left(\int_\Omega|\tilde{T}|^4\mathrm{d}x\mathrm{d}y\mathrm{d}z\right)^{1/4} \leqslant \\
&C_0^2\|\tilde{\boldsymbol{u}}\|^{1/2}_{L^2(\Omega)}\|\nabla\tilde{\boldsymbol{u}}\|^{1/2}_{L^2(\Omega)}\|\nabla T_2\|_{L^2(\Omega)}\|\tilde{T}\|^{1/2}_{L^2(\Omega)}\|\nabla\tilde{T}\|^{1/2}_{L^2(\Omega)} \leqslant \\
&\epsilon_5C_0^2\|\nabla\tilde{\boldsymbol{u}}\|^2_{L^2(\Omega)}+\frac{C_0^2\rho_{10}(t)}{8\epsilon_5\epsilon_6}\|\tilde{T}\|^2_{L^2(\Omega)}+\frac{C_0^2\epsilon_6\rho_{10}(t)}{8\epsilon_5}\|\nabla\tilde{T}\|^2_{L^2(\Omega)},
\end{aligned} \tag{39}
$$

$$
\int_\Omega\left(\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right)\frac{\partial T_2}{\partial z}\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z \leqslant
$$

$$\epsilon_7\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\frac{C_0^2\rho_9(t)\rho_{10}(t)}{8\epsilon_7\epsilon_8}\|\tilde{T}\|_{L^2(\Omega)}^2+\frac{C_0^2\epsilon_8}{8\epsilon_7}\|\nabla\tilde{T}\|_{L^2(\Omega)}^2, \tag{40}$$

$$-\int_\Omega\frac{abP}{p(s)}\tilde{q}\int_0^z\nabla\cdot\boldsymbol{u}_2(x,y,s,t)\,\mathrm{d}s\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z\leqslant$$
$$\frac{CC_0}{2}\rho_{10}(t)\left[\epsilon_9\|\tilde{q}\|_{L^2(\Omega)}^2+\frac{1}{4\epsilon_9}\|\nabla\tilde{q}\|_{L^2(\Omega)}^2+\right.$$
$$\left.\epsilon_{10}\|\tilde{T}\|_{L^2(\Omega)}^2+\frac{1}{4\epsilon_{10}}\|\nabla\tilde{T}\|_{L^2(\Omega)}^2\right]. \tag{41}$$

又因为

$$-\alpha_1\int_M\tilde{T}^2\mathrm{d}x\mathrm{d}y-\tilde{\alpha}\int_M T_2\tilde{T}\mathrm{d}x\mathrm{d}y\leqslant$$
$$-\alpha_1\int_M|\tilde{T}|^2\mathrm{d}x\mathrm{d}y+\tilde{\alpha}\int_M|T_2||\tilde{T}|\,\mathrm{d}x\mathrm{d}y\leqslant$$
$$-\alpha_1\|\tilde{T}\|_{L^2(M)}^2+\frac{\tilde{\alpha}^2}{4\alpha_1}\|T_2\|_{L^2(M)}^2+\alpha_1\|\tilde{T}\|_{L^2(M)}^2\leqslant$$
$$\frac{\tilde{\alpha}^2}{4\alpha_1}\|T_2\|_{L^2(M)}^2. \tag{42}$$

利用分部积分,可得

$$\int_\Omega\left(\int_0^z\frac{bP}{p(s)}\nabla[(1+aq_1)\tilde{T}]\mathrm{d}s\right)\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z=$$
$$-\int_\Omega\left(\int_0^z\frac{bP}{p(s)}[(1+aq_1)\tilde{T}]\mathrm{d}s\right)\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}x\mathrm{d}y\mathrm{d}z=$$
$$\frac{bP}{p(s)}\left[(1+aq_1)\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}\mathrm{d}s\right]\cdot\tilde{T}\mathrm{d}x\mathrm{d}y\mathrm{d}z. \tag{43}$$

将估计式(36)~(38)代入式(31),将式(39)~(42)代入式(32),并应用式(43),两式相加计算整理,得

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\|\tilde{T}\|_{L^2(\Omega)}^2)+\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+$$
$$\|\partial_z\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\|\nabla\tilde{T}\|_{L^2(\Omega)}^2+\|\partial_z\tilde{T}\|_{L^2(\Omega)}^2\leqslant$$
$$\left[k\left(1+\frac{\delta}{4}\right)\sqrt{\rho_{10}(t)}+\frac{C_0^2\rho_8(t)\rho_{10}(t)}{8\epsilon_1\epsilon_2}\right]\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+$$
$$\left[\frac{3}{4}\delta^{-1/3}\sqrt{\rho_{10}(t)}+C\epsilon_1+\frac{C_0^2\epsilon_2}{8\epsilon_1}+C\epsilon_3+C_0{}^2\epsilon_5+\epsilon_7\right]\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+$$
$$\left[\frac{C_0^2\rho_{10}(t)}{8\epsilon_5\epsilon_6}+\frac{C_0^2\rho_9(t)\rho_{10}(t)}{8\epsilon_7\epsilon_8}+\frac{CC_0}{2}\rho_{10}(t)\epsilon_{10}\right]\|\tilde{T}\|_{L^2(\Omega)}^2+$$
$$\left[\frac{C_0^2\epsilon_6\rho_{10}(t)}{8\epsilon_5}+\frac{C_0^2\epsilon_8}{8\epsilon_7}+\frac{CC_0}{8\epsilon_{10}}\rho_{10}(t)\right]\|\nabla\tilde{T}\|_{L^2(\Omega)}^2+$$
$$\left[\frac{C_0^2}{8\epsilon_3\epsilon_4}+\frac{CC_0}{2}\rho_{10}(t)\epsilon_9\right]\|\tilde{q}\|_{L^2(\Omega)}^2+$$

$$\left[\frac{C_0^2\epsilon_4}{8\epsilon_3}\rho_3(t)\rho_{10}(t)+\frac{CC_0\rho_{10}(t)}{8\epsilon_9}\right]\|\nabla\tilde{q}\|_{L^2(\Omega)}^2+\frac{\tilde{\alpha}^2}{4\alpha_1}\|T_2\|_{L^2(M)}^2. \tag{44}$$

将方程(36)与 $\tilde{q}$ 在 $L^2(\Omega)$ 中做内积,通过分部积分,由边界条件(27)~(29),计算得

$$\frac{1}{2}\frac{d}{dt}\|\tilde{q}\|_{L^2(\Omega)}^2+\|\nabla\tilde{q}\|_{L^2(\Omega)}^2+\|\partial_z\tilde{q}\|_{L^2(\Omega)}^2=$$
$$-\int_\Omega\left[(\tilde{\boldsymbol{u}}\cdot\nabla)q_2-\left(\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}ds\right)\frac{\partial q_2}{\partial z}\right]\cdot\tilde{q}dxdydz-$$
$$\int_M(\beta_1\tilde{q}+\tilde{\beta}q_2)\tilde{q}dxdy. \tag{45}$$

类似式(36)~(38)的计算方法,可得如下估计:

$$-\int_\Omega(\tilde{\boldsymbol{u}}\cdot\nabla)q_2\cdot\tilde{T}dxdydz\leqslant$$
$$\epsilon_{11}C_0^2\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\frac{\rho_{10}(t)}{8\epsilon_{11}\epsilon_{12}}\|\tilde{q}\|_{L^2(\Omega)}^2+\frac{\epsilon_{12}\rho_{10}(t)}{8\epsilon_{11}}\|\nabla\tilde{q}\|_{L^2(\Omega)}^2, \tag{46}$$

$$\int_\Omega\left(\int_0^z\nabla\cdot\tilde{\boldsymbol{u}}ds\right)\frac{\partial q_2}{\partial z}\tilde{q}dxdydz\leqslant$$
$$\epsilon_{13}\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\frac{C_0^2\rho_{10}(t)}{8\epsilon_{13}\epsilon_{14}}\|\tilde{q}\|_{L^2(\Omega)}^2+\frac{\epsilon_{14}}{8\epsilon_{13}}\|\nabla\tilde{q}\|_{L^2(\Omega)}^2, \tag{47}$$

其中

$$\int_M(\beta_1\tilde{q}+\tilde{\beta}q_2)\tilde{q}dxdy\leqslant\frac{\tilde{\beta}^2}{4\beta_1}\|q_2\|_{L^2(M)}^2. \tag{48}$$

将式(46)~(48)代入式(45),得

$$\frac{1}{2}\frac{d}{dt}\|\tilde{q}\|_{L^2(\Omega)}^2+\|\nabla\tilde{q}\|_{L^2(\Omega)}^2+\|\partial_z\tilde{q}\|_{L^2(\Omega)}^2\leqslant$$
$$(\epsilon_{11}C_0^2+\epsilon_{13})\|\nabla\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\left(\frac{\rho_{10}(t)}{8\epsilon_{11}\epsilon_{12}}+\frac{C_0^2\rho_{10}(t)}{8\epsilon_{13}\epsilon_{14}}\right)\|\tilde{q}\|_{L^2(\Omega)}^2+$$
$$\left(\frac{\epsilon_{12}\rho_{10}(t)}{8\epsilon_{11}}+\frac{\epsilon_{13}}{8\epsilon_{14}}\right)\|\nabla\tilde{q}\|_{L^2(\Omega)}^2+\frac{\tilde{\beta}^2}{4\beta_1}\|q_2\|_{L^2(M)}^2. \tag{49}$$

我们取

$$\epsilon_1=\epsilon_7=\epsilon_{13}=\frac{1}{8},\ \epsilon_2=\epsilon_5=\epsilon_{11}=\frac{1}{8{C_0}^2},\ \epsilon_3=\frac{1}{8C},$$
$$\epsilon_4=\frac{1}{4C{C_0}^2\rho_3(t)\rho_{10}(t)},\ \epsilon_6=\frac{1}{3{C_0}^3\rho_{10}(t)},\ \epsilon_8=\frac{1}{3{C_0}^2},$$
$$\epsilon_9=\frac{1}{2CC_0\rho_{10}(t)},\ \epsilon_{10}=\frac{3C{C_0}^2\rho_{10}(t)}{8},\ \epsilon_{12}=\frac{1}{4C_0\rho_{10}(t)},\ \epsilon_{14}=\frac{1}{16}.$$

将式(44)、(49)相加,计算得

$$\frac{d}{dt}(\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\|\tilde{T}\|_{L^2(\Omega)}^2+\|\tilde{q}\|_{L^2(\Omega)}^2)\leqslant$$
$$\left[2k\left(1+\frac{\delta}{4}\right)\sqrt{\rho_{10}(t)}+\frac{C_0^2\rho_8(t)\rho_{10}(t)}{4\epsilon_1\epsilon_2}\right]\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+$$

$$\left[\frac{C_0^2\rho_{10}(t)}{4\epsilon_5\epsilon_6}+\frac{C_0^2\rho_9(t)\rho_{10}(t)}{4\epsilon_7\epsilon_8}+CC_0\rho_{10}(t)\epsilon_{10}\right]\|\tilde{T}\|_{L^2(\Omega)}^2+$$
$$\left[\frac{C_0^2}{4\epsilon_3\epsilon_4}+CC_0\rho_{10}(t)\epsilon_9+\frac{\rho_{10}(t)}{4\epsilon_{11}\epsilon_{12}}+\frac{C_0^2\rho_{10}(t)}{4\epsilon_{13}\epsilon_{14}}\right]\|\tilde{q}\|_{L^2(\Omega)}^2+$$
$$\frac{\tilde{\alpha}^2}{2\alpha_1}\|T_2\|_{L^2(M)}^2+\frac{\tilde{\beta}^2}{2\beta_1}\|q_2\|_{L^2(M)}^2. \tag{50}$$

由 Gronwall 不等式,可得

$$\|\tilde{\boldsymbol{u}}\|_{L^2(\Omega)}^2+\|\tilde{T}\|_{L^2(\Omega)}^2+\|\tilde{q}\|_{L^2(\Omega)}^2\leqslant$$
$$e^{\int_0^t m(s)\mathrm{d}s}\left[\frac{\tilde{\alpha}^2}{2\alpha_1}\int_0^t\|T_2(s)\|_{L^2(M)}^2\mathrm{d}s+\frac{\tilde{\beta}^2}{2\beta_1}\int_0^t\|q_2(s)\|_{L^2(M)}^2\mathrm{d}s\right],$$

其中

$$m(t)=\max\left\{2k\left(1+\frac{\delta}{4}\right)\sqrt{\rho_{10}(t)}+\frac{C_0^2\rho_8(t)\rho_{10}(t)}{4\epsilon_1\epsilon_2},\right.$$
$$\frac{C_0^2\rho_{10}(t)}{4\epsilon_5\epsilon_6}+\frac{C_0^2\rho_9(t)\rho_{10}(t)}{4\epsilon_7\epsilon_8}+CC_0\rho_{10}(t)\epsilon_{10},$$
$$\left.\frac{C_0^2}{4\epsilon_3\epsilon_4}+CC_0\rho_{10}(t)\epsilon_9+\frac{\rho_{10}(t)}{4\epsilon_{11}\epsilon_{12}}+\frac{C_0^2\rho_{10}(t)}{4\epsilon_{13}\epsilon_{14}}\right\}.$$

于是定理 1 得证.

3 结　　论

本文主要展示了如何控制水汽比、利用能量估计的办法,得到了湿大气原始方程组对黏性系数的连续依赖性.接下来,也可以继续研究湿大气原始方程组的收敛性,即当方程组的系数趋近于零时所产生的影响.据笔者所知,目前这类研究在文献中尚未出现,而且这类研究还可以向带随机力的原始方程组、海洋原始方程组、大气原始方程组以及耦合了海洋和大气的原始方程组甚至干大气原始方程组展开.我们希望本文的研究能为读者带来一定的灵感,这也是我们下一步研究的一个重点方向.

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Continuous Dependence on Boundary Parameters of the Original Equations for Large-Scale Wet Atmosphere

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Abstract: Large-scale dynamic equations for atmosphere are controlled by the original equations derived from the Navier-Stokes equations, and coupled with the thermodynamics and salinity diffusion transport equations. In the past few decades, the atmosphere, ocean, and atmosphere-ocean coupling original equations were extensively studied from the perspective of mathematics. The previous literatures mainly focused on the mathematical logic or well-posedness of the original equations. The stability of the original equations was addressed. Given the inevitable errors in the model establishment and simplification, the effects of coefficients' small changes on solutions' great changes were studied for the original equations. Prior estimates of the solutions, combined with energy estimation and the differential inequality technique, were used to control steam ratios. The results prove the continuous dependence of the solutions to the large-scale wet atmosphere original equations on boundary parameters.

Key words: original equation; prior estimation; continuous dependence

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