

生态位贴近度的 Type-2 直接 T-S 模糊控制*

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摘要: 生态位在生态系统中起着重要作用, type-2 直接 T-S 模糊控制在稳定一类具有参数不确定性的非线性系统时, 具有很好的可操作性, 将系统的生物个体的进化特性、自适应性与直接 T-S 模糊 type-2 控制方法相结合, 以生态位贴近度函数作为 type-2 T-S 模糊控制的后果, 构造具有生物特性的 type-2 直接 T-S 模糊控制方法, 求得生态位生态因子的自适应律, 反映了生物个体的自适应利用环境的程度, 运用 Lyapunov 方法分析系统稳定性, 并通过实例仿真对比分析得出 type-2 在稳定性和收敛性上是优于 type-1 的, 此方法的提出有利于环境的和谐和生态系统的稳定及生态环境的可持续发展, 也使模糊控制有了实际的物理背景。

关键词: 生态位; type-2; 模糊控制; 贴近度; 可持续发展

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引言

生态文明建设对生态环境的要求越来越高, 近年来经济的快速发展带来环境的恶化和生态环境的破坏, 制约着经济的进一步发展和影响着居民的身心健康. 生态环境受到人们越来越多的重视. 可持续的生态环境^[1]是我们追求的目标, 而复杂生态系统中很多重要的生态学理论问题都是以生态位^[2]为基础单位的, 生态位概念已遍及整个生态学科. 生态位及其相关概念的量化和模型化是生态系统的根本问题. 而生态位贴近度正是量化生态位的工具, 因此, 生态系统建模^[3-4]和生态位模糊建模^[5]对生态系统的非线性分析特性^[6]、仿真识别和量化发展工作具有重要的意义. 但这些只是存在理论意义, 缺乏实际的生态位背景和生物个体自适应的效果体现. 所以, 将生物系统的非线性进化特性自适应性、容错性和稳定性与控制方法结合^[7-9], 形成具有生物特性模糊控制方法^[10]和系统故障识别方法, 可以使生态环境系统逐步稳定达到破坏最小化, 有利于生态自然环境的和谐和生态系统的可持续发展. 这些方法都是 type-1 模糊控制, 在稳定性、抗干扰性及收敛性方面难以达到理想效果, 且限制了描述生态系统的不确定性

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和生态位生物个体的自组织、自学习、自适应特性。

Type-1 型 T-S 模糊系统具有强大的非线性系统辨识和控制能力,在保证系统稳定性和逼近性方面具有很大的优势^[7],操作实现简单,解决了控制领域的许多问题^[11]。然而,在一定程度上,模糊集的隶属函数限制了模糊集描述不确定性的能力,使其描述不确定性的能力受到了限制,同时噪音也影响系统的效果。由于 type-1 隶属度函数是精确的,不能描述更为复杂的模糊问题,导致其在应用上受限。Type-2 可以处理隶属度函数是模糊的问题,扩展了 type-1 的应用范围,具有更强的处理模糊问题的能力。Type-2 模糊集合将 type-1 模糊集合中的精确隶属度值模糊化,是 type-1 模糊集的扩展,因此其描述实际系统不确定性的能力得到了增强,具有更强的抗干扰和处理不确定问题的能力^[12]。同时,type-2 模糊集将区间 2 型模糊集作为系统的输入或输出,降低了 type-2 模糊集的计算复杂度,提高了 type-2 模糊系统的实时应用能力,促进了 type-2 模糊集理论的广泛应用。T-S type-2 型模糊模型控制方法在保证闭环系统的稳定性、收敛性、解决数据的不确定性和噪声对系统的影响方面有较好的效果。它不仅具有 2 型模糊集处理的特点,而且可以减少规则不确定性对系统的影响。同时,具 T-S 模糊模型输入变量的线性组合特性^[13],可以改进系统建模准确性,减少系统中规则的数量等。

非线性、时变及纯滞后系统的控制等,日益受到人们的重视,并成功地应用于很多领域^[10-11]。Type-2 模糊系统具有很强的抗干扰能力,许多学者研究具有类似生物特性的模糊控制方法^[12-14],在控制过程中模仿生物的功能和自适应能力,不但能够主动适应外界环境的变化,还能开发利用环境始终朝着有利于自身的方向发展^[15]。此模糊系统不但具备模糊系统的抗干扰性,还将生物个体的自适应性融入到模糊控制过程中,提高了系统的智能特性。但是由于多维超体积模糊生态位等原因不易确定,实际中很难得到有效结果。生态位是生态系统的基本单元,将生物个体具备的自适应、自组织、自学习能力融入到 type-1 模糊系统的设计中,构造具有生物特性的模糊系统。该系统除了具备常规模糊系统的自适应性外,还具有系统随环境变化的自我调节能力以及可移植性的优点。但是鉴于隶属度以上的缺点,无法求出生态位中生物个体的自适应率。而生态位贴切度恰好反映生物个体在生态系统中适应的情况,直接型 T-S 模糊控制具有操作简单,控制目标更容易识别的特点。所以本文提出了将生态系统的生物个体的进化性、自适应性以及稳定性的生态位与 type-2 T-S 模糊控制^[16-17]相结合,构造具有生物特性的直接 T-S 模糊控制方法,生态位的贴切度函数作为 type-2 直接 T-S 模糊控制的后件。通过推理求出生态因子的自适应率,并调整系统的状态,反映了生物个体的适应、发展能力和开发、利用环境的能力,朝着有利于自身发展的方向,可以使生态系统稳定向前发展和达到最佳的平衡稳定状态,有利于环境的和谐、生态系统的稳定和可持续发展。

1 生态位控制

考虑如下 n 阶非线性系统:

$$\begin{cases} \dot{x}_i = x_{i+1}, & 1 \leq i \leq n-1, \\ x^{(n)} = f(x, \dot{x}, \dots, x^{(n-1)}) + bu, \\ y = x_1, \end{cases} \quad (1)$$

其中 $u \in \mathbf{R}$ 和 $y \in \mathbf{R}$ 分别是系统的输入和输出, $\mathbf{X} = (x, \dot{x}, \dots, x^{(n-1)}) = (x_1, x_2, \dots, x_n)^T \in \mathbf{R}^n$ 是通过测量可以得到的系统状态向量, b 为常数, $f(x, \dot{x}, \dots, x^{(n-1)})$ 为未知的非线性连续函数且假设满足 Lipschitz 条件,保证系统解的存在唯一性。

1.1 直接 T-S 模糊系统的描述

由于 $f(x, \dot{x}, \dots, x^{(n-1)})$ 是未知的, 采用区间 type-2 直接 T-S 模糊系统来逼近它. 将生态系统的生态位作为 type-2 直接 T-S 模糊控制的模糊规则前件, 生态位贴近度函数作为模糊规则的后件, type-2 直接 T-S 模糊规则描述为

R_u^k : 如果 x_1^k 是 A_1^k , 且 x_2^k 是 A_2^k, \dots , 且 x_n^k 是 A_n^k , 则

$$\tilde{u}^k = a_0^k = \frac{3}{2} - \phi \left[\frac{\lambda_u^k - \lambda^k}{\sigma_u^k + \sigma} \right], \quad k = 1, 2, \dots, M, \quad (2)$$

这里, A_i^k 是状态向量 x_i^k 的区间 type-2 模糊集, $\lambda_u = (x_1, x_2, \dots, x_n)$ 代表着生态系统中生态位的实际生态因子, $\lambda = (x_1^*, x_2^*, \dots, x_n^*)$ 代表着生态系统中生态位的理想生态因子, σ_u^k 表示实际生态因子隶属函数的宽度. 控制后件表示生态系统实际的生态位和理想的生态位的差别, 这里用 Gauss 隶属函数表示, 满足 $x_i^l - x_j^l \geq \sigma$, x_i^k, x_j^k 和 σ 分别是隶属函数的中心和理想宽度. 这种差别可以用贴近度函数

$$H(A, B) = \frac{3}{2} - \phi \left[\frac{\lambda_u^k - \lambda^k}{\sigma_u^k + \sigma} \right]$$

表示, 这里

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt.$$

通过贴近度函数, 我们知道后件是一个零阶的 T-S 模糊模型. 为方便我们用 a_0^k 代替

$$\frac{3}{2} - \phi \left[\frac{\lambda_u^k - \lambda^k}{\sigma_u^k + \sigma} \right].$$

a_0^k 是 type-2 型模糊系统输出后件常数, 前件为区间 type-2 模糊集.

模糊系统 $\hat{u}(X)$ 的规则前件是一个区间 type-2 型模糊集, 其隶属函数为

$$\rho_{A_1^k, A_2^k, \dots, A_n^k}(X) = \rho_{A_1^k}(x_1) \cdot \rho_{A_2^k}(x_2) \cdot \dots \cdot \rho_{A_n^k}(x_n) = \prod_{l=1}^n \rho_{A_l^k}(x_l), \quad k = 1, 2, \dots, n,$$

其中 Gauss 隶属函数

$$\rho_{A_i^k}(x_i) = \exp \left(- \left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l} \right)^2 \right), \quad i = 1, 2, \dots, n. \quad (3)$$

输出直接 type-2 型模糊集可用 \tilde{Z}_u^i 表示, $i = 1, 2, \dots, M$, 采用平均中心去模糊, 则降型为

$$\tilde{U}(\tilde{Z}_u^1, \tilde{Z}_u^2, \dots, \tilde{Z}_u^M) = \int_{z_u^1} \int_{z_u^2} \dots \int_{z_u^M} \tau_{i=1}^M \rho_{\tilde{Z}_i^i}(\tilde{z}_i^i) \left/ \frac{\sum_{i=1}^M z_u^i \hat{u}^i(X)}{\sum_{i=1}^M z_u^i} \right. \quad (4)$$

对 A_i^k 使用 type-2 型隶属函数, 则式(4)为

$$\hat{U}(\tilde{Z}_u^1, \tilde{Z}_u^2, \dots, \tilde{Z}_u^M) = \int_{z_u^1} \int_{z_u^2} \dots \int_{z_u^M} 1 \left/ \frac{\sum_{i=1}^M z_u^i \hat{u}^i(X)}{\sum_{i=1}^M z_u^i} \right., \quad (5)$$

其中

$$\hat{U} = [\hat{u}_l, \hat{u}_r], z_u^i \in \tilde{Z}_u^i \in [z_u^i, \bar{z}_u^i], z_u^i = \rho_{A_1^i}(x_1) \cdot \rho_{A_2^i}(x_2) \cdots \rho_{A_n^i}(x_n),$$

$$\bar{z}_u^i = \rho_{\bar{A}_1^i}(x_1) \cdot \rho_{\bar{A}_2^i}(x_2) \cdots \rho_{\bar{A}_n^i}(x_n),$$

“.”是取小或乘积 t - 范式。

利用 KM 迭代法^[7]可知

$$\hat{u}_l = \frac{\sum_{i=1}^L \bar{z}_u^i \hat{u}^i + \sum_{j=L+1}^M \bar{z}_u^j \hat{u}^j}{\sum_{i=1}^L \bar{z}_u^i + \sum_{j=L+1}^M \bar{z}_u^j} = \sum_{i=1}^L q_{ul}^i \hat{u}^i + \sum_{j=L+1}^M q_{ul}^j \hat{u}^j =$$

$$\sum_{i=1}^L q_{ul}^i \theta_{ul}^i X + \sum_{j=L+1}^M q_{ul}^j \theta_{ul}^j X = (\underline{Q}_{ul} \quad \bar{Q}_{ul}) \begin{pmatrix} \underline{\theta}_{ul} \\ \bar{\theta}_{ul} \end{pmatrix} X = \xi_{ul} \theta_{ul} X, \quad (6)$$

式中

$$q_{ul}^i = \frac{\bar{z}_u^i}{\sum_{i=1}^L \bar{z}_u^i + \sum_{j=L+1}^M \bar{z}_u^j}, q_{ul}^j = \frac{\bar{z}_u^j}{\sum_{i=1}^L \bar{z}_u^i + \sum_{j=L+1}^M \bar{z}_u^j}, \underline{Q}_{ul} = (q_{ul}^1, q_{ul}^2, \dots, q_{ul}^L),$$

$$\bar{Q}_{ul} = (q_{ul}^{L+1}, q_{ul}^{L+2}, \dots, q_{ul}^M), \underline{\theta}_{ul} = (\theta_{ul}^1, \theta_{ul}^2, \dots, \theta_{ul}^L)^T, \bar{\theta}_{ul} = (\theta_{ul}^{L+1}, \theta_{ul}^{L+2}, \dots, \theta_{ul}^M)^T,$$

$$\xi_{ul} = (\underline{Q}_{ul} \quad \bar{Q}_{ul}), \theta_{ul}^i = (a_{i0}^i, 0, \dots, 0) \in R^{1 \times (n+1)}, \quad i = 1, 2, \dots, L,$$

$$\theta_{ul}^j = (a_{j0}^j, 0, \dots, 0) \in R^{1 \times (n+1)}, \quad j = L+1, L+2, \dots, M, X = (1, x_1, \dots, x_n)^T,$$

$$\hat{u}_r = \frac{\sum_{i=1}^R \bar{z}_u^i \hat{u}^i + \sum_{j=R+1}^M \bar{z}_u^j \hat{u}^j}{\sum_{i=1}^R \bar{z}_u^i + \sum_{j=R+1}^M \bar{z}_u^j} = \sum_{i=1}^R q_{ur}^i \hat{u}^i + \sum_{j=R+1}^M q_{ur}^j \hat{u}^j =$$

$$\sum_{i=1}^R q_{ur}^i \theta_{ur}^i X + \sum_{j=R+1}^M q_{ur}^j \theta_{ur}^j X = (\underline{Q}_{ur} \quad \bar{Q}_{ur}) \begin{pmatrix} \underline{\theta}_{ur} \\ \bar{\theta}_{ur} \end{pmatrix} X = \xi_{ur} \theta_{ur} X, \quad (7)$$

其中

$$q_{ur}^i = \frac{\bar{z}_u^i}{\sum_{i=1}^R \bar{z}_u^i + \sum_{j=R+1}^M \bar{z}_u^j}, q_{ur}^j = \frac{\bar{z}_u^j}{\sum_{i=1}^R \bar{z}_u^i + \sum_{j=R+1}^M \bar{z}_u^j}, \underline{Q}_{ur} = (q_{ur}^1, q_{ur}^2, \dots, q_{ur}^R),$$

$$\bar{Q}_{ur} = (q_{ur}^{R+1}, q_{ur}^{R+2}, \dots, q_{ur}^M), \underline{\theta}_{ur} = (\theta_{ur}^1, \theta_{ur}^2, \dots, \theta_{ur}^R)^T, \bar{\theta}_{ur} = (\theta_{ur}^{R+1}, \theta_{ur}^{R+2}, \dots, \theta_{ur}^M)^T,$$

$$\xi_{ur} = (\underline{Q}_{ur} \quad \bar{Q}_{ur}), \theta_{ur}^i = (a_{i0}^i, 0, \dots, 0) \in R^{1 \times (n+1)}, \quad i = 1, 2, \dots, R,$$

$$\theta_{ur}^j = (a_{j0}^j, 0, \dots, 0) \in R^{1 \times (n+1)}, \quad j = R+1, R+2, \dots, M,$$

$$X = (1, x_1, \dots, x_n)^T, a_0^i = (a_{i0}^i \quad a_{i0}^i)^T, a_0^j = (a_{j0}^j \quad a_{j0}^j)^T, a_0 = (a_0^i \quad a_0^i)^T.$$

采用中心平均去模糊,由式(6)和(7)可得此 type-2 型模糊系统输出为

$$\hat{u}(X | \theta_{ul}, \theta_{ur}) = \frac{\hat{u}_l + \hat{u}_r}{2} = \frac{1}{2} (\xi_{ul} \theta_{ul} + \xi_{ur} \theta_{ur}) X = \frac{1}{2} \xi_u \theta_u, \quad (8)$$

其中

$$\xi_u = (\xi_{ul} \quad \xi_{ur}) X, \theta_u = (\theta_{ul} \quad \theta_{ur})^T.$$

1.2 生态位贴近度 T-S 模型参数优化方法

借用基于生态位贴近度 type-1 直接模糊 T-S 模型参数优化的反向传播算法的方法^[1]优化生态位贴近度 type-2 直接模糊 T-S 模型的参数,已知反向传播算法的输入、输出数据为 (\mathbf{x}_0^p, y_0^p) , $\mathbf{x}_0^p \in U \in \mathbf{R}^n, y_0^p \in V \in \mathbf{R}$,任务是确定形如式(2)的生态位贴近度 T-S 模型参数,使优化后的输出误差

$$e^p = \frac{1}{2} [\hat{u}(\mathbf{x}_0^p) - y_0^p]^2$$

最小.假设 M 已知,通过调整 $\lambda_u^l, \sigma_u^l, x_i^l, \sigma_i^l$ 使 e^p 最小.为讨论方便,用 $\hat{e}, \hat{u}, \hat{y}$ 代替 $e^p, \hat{u}(\mathbf{x}_0^p)$ 和 y_0^p .此时

$$\begin{aligned} \lambda_u^i &= (\lambda_{ul}^i, \lambda_{ur}^i), \sigma_u^i = (\sigma_{ul}^i, \sigma_{ur}^i), x_i^l = (x_{il}^l, x_{ir}^l), \sigma_i^l = (\sigma_{il}^l, \sigma_{ir}^l), \\ \lambda_u^j &= (\lambda_{ul}^j, \lambda_{ur}^j), \sigma_u^j = (\sigma_{ul}^j, \sigma_{ur}^j), x_i^j = (x_{il}^j, x_{ir}^j), \sigma_i^j = (\sigma_{il}^j, \sigma_{ir}^j). \end{aligned}$$

用梯度下降法优化参数^[7]

$$\lambda_{ul}^i(\eta + 1) = \lambda_{ul}^i(\eta) - \gamma \left. \frac{\partial e}{\partial \lambda_{ul}^i} \right|_{\eta}, \quad i = 1, 2, \dots, L, \eta = 1, 2, \dots,$$

γ 是确定的步长.

这时

$$\frac{\partial \hat{e}}{\partial \lambda_{ul}^i} = \frac{\partial \hat{e}}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial a_0} \frac{\partial a_0}{\partial a_0^i} \frac{\partial a_0^i}{\partial \lambda_{ul}^i} \frac{\partial \lambda_{ul}^i}{\partial \lambda_{ul}^i} = (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^i - \lambda)/(\sigma_{ul}^i + \sigma)]^2/2} \frac{1}{\sigma_{ul}^i + \sigma} \frac{1}{D} W^l \mathbf{X}, \quad (9)$$

其中

$$D = \sum_{l=1}^m W^l, \quad W^l = \prod_{i=1}^n \exp\left(-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l}\right)^2\right), \quad a_0 = \sum_{l=1}^m a_0^l W^l.$$

则有

$$\lambda_{ul}^i(\eta + 1) = \lambda_{ul}^i(\eta) - \gamma (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^i - \lambda)/(\sigma_{ul}^i + \sigma)]^2/2} \frac{1}{\sigma_{ul}^i(\eta) + \sigma} \frac{1}{D} W^l \mathbf{X}, \quad (10)$$

$$\frac{\partial \hat{e}}{\partial \lambda_{ur}^i} = \frac{\partial \hat{e}}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial a_0} \frac{\partial a_0}{\partial a_0^i} \frac{\partial a_0^i}{\partial \lambda_{ur}^i} \frac{\partial \lambda_{ur}^i}{\partial \lambda_{ur}^i} = (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^i - \lambda)/(\sigma_{ur}^i + \sigma)]^2/2} \frac{1}{\sigma_{ur}^i + \sigma} \frac{1}{D} W^l \mathbf{X}, \quad (11)$$

$$\lambda_{ur}^i(\eta + 1) = \lambda_{ur}^i(\eta) - \gamma (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^i - \lambda)/(\sigma_{ur}^i + \sigma)]^2/2} \frac{1}{\sigma_{ur}^i(\eta) + \sigma} \frac{1}{D} W^l \mathbf{X}. \quad (12)$$

于是有

$$\frac{\partial \hat{e}}{\partial \lambda_{ul}^j} = \frac{\partial \hat{e}}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial a_0} \frac{\partial a_0}{\partial a_0^j} \frac{\partial a_0^j}{\partial \lambda_{ul}^j} \frac{\partial \lambda_{ul}^j}{\partial \lambda_{ul}^j} = (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^j - \lambda)/(\sigma_{ul}^j + \sigma)]^2/2} \frac{1}{\sigma_{ul}^j + \sigma} \frac{1}{D} W^l \mathbf{X}, \quad (13)$$

可得

$$\lambda_{ul}^j(\eta + 1) = \lambda_{ul}^j(\eta) - \gamma (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^j - \lambda)/(\sigma_{ul}^j + \sigma)]^2/2} \frac{1}{\sigma_{ul}^j(\eta) + \sigma} \frac{1}{D} W^l \mathbf{X}, \quad (14)$$

$$\lambda_{ur}^j(\eta + 1) = \lambda_{ur}^j(\eta) - \gamma (\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^j - \lambda)/(\sigma_{ur}^j + \sigma)]^2/2} \frac{1}{\sigma_{ur}^j(\eta) + \sigma} \frac{1}{D} W^l \mathbf{X}. \quad (15)$$

同理可得

$$\begin{cases} \sigma_{ul}^i(\eta + 1) = \sigma_{ul}^i(\eta) - \gamma(\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^i - \lambda)/(\sigma_{ul}^i + \sigma)]^2/2} \frac{\lambda_{ul}^i - \lambda}{(\sigma_{ul}^i(\eta) + \sigma)^2} \frac{1}{D} W^l X, \\ \sigma_{ul}^j(\eta + 1) = \sigma_{ul}^j(\eta) - \gamma(\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^j - \lambda)/(\sigma_{ul}^j + \sigma)]^2/2} \frac{\lambda_{ul}^j - \lambda}{(\sigma_{ul}^j(\eta) + \sigma)^2} \frac{1}{D} W^l X, \\ \sigma_{ur}^i(\eta + 1) = \sigma_{ur}^i(\eta) - \gamma(\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^i - \lambda)/(\sigma_{ur}^i + \sigma)]^2/2} \frac{\lambda_{ur}^i - \lambda}{(\sigma_{ur}^i(\eta) + \sigma)^2} \frac{1}{D} W^l X, \\ \sigma_{ur}^j(\eta + 1) = \sigma_{ur}^j(\eta) - \gamma(\hat{u} - \hat{y}) \frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^j - \lambda)/(\sigma_{ur}^j + \sigma)]^2/2} \frac{\lambda_{ur}^j - \lambda}{(\sigma_{ur}^j(\eta) + \sigma)^2} \frac{1}{D} W^l X. \end{cases} \quad (16)$$

同理,可调整参数 $x_{ul}^l, x_{ur}^l, x_{ul}^j, x_{ur}^j, \sigma_{ul}^l, \sigma_{ur}^l, \sigma_{ul}^j, \sigma_{ur}^j$ 也可以采用同样的方法优化.

2 生态位贴适度 type-2 直接模糊控制器

对系统(1)选择如下控制器:

$$u^* = \frac{1}{b} [-f(X) + y_m^{(n)} + \mathbf{K}^T \mathbf{e}], \quad (17)$$

其中 $\mathbf{e} = \mathbf{y}_m - \mathbf{y}$, \mathbf{y}_m 为一个在紧集下有界的参考信号, $\mathbf{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$, $\mathbf{K} = (k_n, k_{n-1}, \dots, k_1)^T$, 由式(1)知, 当 $t \rightarrow \infty$ 时, $e(t) \rightarrow 0$. 下面用 type-2 直接模糊控制器 $u_D(X | \boldsymbol{\theta}_D)$, 其中 $\boldsymbol{\theta}_D = [\theta_{Dl}, \theta_{Dr}]$, 代替式(17)中的 u , 得到

$$x^{(n)} = f + b u_D(X | \boldsymbol{\theta}_D). \quad (18)$$

把式(17)代入式(18), 得到

$$x^{(n)} = y_m^{(n)} + \mathbf{K}^T \mathbf{e} + b [u_D(X | \boldsymbol{\theta}_D) - u^*], \quad (19)$$

即

$$e^{(n)} = -\mathbf{K}^T \mathbf{e} + b [u^* - u_D(X | \boldsymbol{\theta}_D)]. \quad (20)$$

令

$$\mathbf{A}_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1 \end{pmatrix}, \quad \mathbf{b}_c = [0, \dots, b]^T \in R^n,$$

则得到

$$\dot{\mathbf{e}} = \mathbf{A}_c \mathbf{e} + \mathbf{b}_c [u^* - u_D(X | \boldsymbol{\theta}_D)]. \quad (21)$$

由于 $|s\mathbf{I} - \mathbf{A}_c| = h(s)$ 是 Hurwitz 多项式, 因此可知 \mathbf{A}_c 是稳定矩阵, 则必然存在唯一 n 阶正定矩阵 \mathbf{P} 使得 Lyapunov 方程 $\mathbf{A}_c^T \mathbf{P} + \mathbf{P} \mathbf{A}_c = -\mathbf{Q}$, \mathbf{Q} 为任意 n 阶正定矩阵.

构造 Lyapunov 函数: $V_e = \frac{1}{2} \mathbf{e}^T \mathbf{P} \mathbf{e}$, 则得

$$\dot{V}_e = \frac{1}{2} (\dot{\mathbf{e}}^T \mathbf{P} \mathbf{e} + \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}}) = -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \mathbf{e}^T \mathbf{P} \mathbf{b}_c [u^* - u_D(X | \boldsymbol{\theta}_D)]. \quad (22)$$

令 V_M 为一给定的正常数,可得:1) 当 $V_e \leq V_M$ 时,知 V_e 有界;2) 当 $V_e > V_M$ 时,由文献[13]知, V_e 同样有界.

3 自适应律的设计

在模糊逻辑系统中为了调整参数,因此,最优参数 $\theta_{Dl}^*, \theta_{Dr}^*$ 定义如下:

$$\begin{cases} \theta_{Dl}^* = \arg \min_{\theta_{Dl} \in \Omega_D} \left[\sup_{X \in \Omega_X, \bar{X} \in \Omega_{\bar{X}}} |u^*(X) - u_l(\bar{X} | \theta_{Dl})| \right], \\ \theta_{Dr}^* = \arg \min_{\theta_{Dr} \in \Omega_D} \left[\sup_{X \in \Omega_X, \bar{X} \in \Omega_{\bar{X}}} |u^*(X) - u_r(\bar{X} | \theta_{Dr})| \right], \end{cases} \quad (23)$$

此处 Ω_D, Ω_X 和 $\Omega_{\bar{X}}$ 是边界紧集数,

$$\Omega_D = \{ \theta_{Dl}^i, \theta_{Dr}^i : \|\theta_{Dl}^i\| \leq M_D, \|\theta_{Dr}^i\| \leq M_D \},$$

$$\Omega_X = \{ X : \|X\| \leq M_X \}, \quad \Omega_{\bar{X}} = \{ \bar{X} : \|\bar{X}\| \leq M_{\bar{X}} \},$$

这里 M_D, M_X 和 $M_{\bar{X}}$ 都为正常数.定义最小逼近误差如下:

$$\omega = u_D(X | \theta_{Dl}^*, \theta_{Dr}^*) - u^*, \quad (24)$$

则得到误差方程

$$\begin{aligned} \dot{e} &= A_c e + b_c [u^* - u_D(X | \theta_D)] = \\ &= A_c e + b_c [u_D(X | \theta_D^*) - u_D(X | \theta_D)] - b_c \omega, \end{aligned} \quad (25)$$

$$u(X | \theta_D) - u(X | \theta_D^*) = \frac{1}{2} \left(\sum_{i=1}^{L^2} q_{ul}^i \Theta_{ul}^i + \sum_{j=L^2+1}^M q_{ul}^j \Theta_{ul}^j + \sum_{i=1}^{R^2} q_{ur}^i \Theta_{ur}^i + \sum_{j=R^2+1}^M q_{ur}^j \Theta_{ur}^j \right) X,$$

其中

$$\Theta_{ul}^i = \theta_{ul}^i - \theta_{ul}^{i*}, \quad \Theta_{ul}^j = \theta_{ul}^j - \theta_{ul}^{j*}, \quad \Theta_{ur}^i = \theta_{ur}^i - \theta_{ur}^{i*}, \quad \Theta_{ur}^j = \theta_{ur}^j - \theta_{ur}^{j*}.$$

将式(8)代入式(25)得

$$\dot{e} = A_c e + \frac{1}{2} b_c \xi_u [\theta_D^* - \theta_D] - b_c \omega. \quad (26)$$

现在考虑 Lyapunov 函数

$$V = \frac{1}{2} e^T P e + \frac{1}{2 \Gamma_u} (\theta_D^* - \theta_D)^T (\theta_D^* - \theta_D), \quad (27)$$

把式(25)和(26)代入式(27),计算并求导,得到

$$\begin{aligned} \dot{V} &= -\frac{1}{2} e^T Q e - e^T P b_c \omega + \\ & \left[\sum_{i=1}^{R^2} \Theta_{ul}^i (\Gamma_{ul}^i)^{-1} \left((\dot{\theta}_{ul}^i)^T + \frac{1}{2} \Gamma_{ul}^i e^T P b_c q_{ul}^i X \right) \right] + \\ & \left[\sum_{j=R^2+1}^M \Theta_{ul}^j (\Gamma_{ul}^j)^{-1} \left((\dot{\theta}_{ul}^j)^T + \frac{1}{2} \Gamma_{ul}^j e^T P b_c q_{ul}^j X \right) \right] + \\ & \left[\sum_{i=1}^{R^2} \Theta_{ur}^i (\Gamma_{ur}^i)^{-1} \left((\dot{\theta}_{ur}^i)^T + \frac{1}{2} \Gamma_{ur}^i e^T P b_c q_{ur}^i X \right) \right] + \\ & \left[\sum_{j=R^2+1}^M \Theta_{ur}^j (\Gamma_{ur}^j)^{-1} \left((\dot{\theta}_{ur}^j)^T + \frac{1}{2} \Gamma_{ur}^j e^T P b_c q_{ur}^j X \right) \right], \end{aligned} \quad (28)$$

其中

$$\Gamma_u = [\Gamma_{ul} \quad \Gamma_{ur}], \Gamma_{ul} = \begin{bmatrix} \tau_{ul0} & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \tau_{uln} \end{bmatrix}.$$

则得自适应律为

$$\begin{cases} \dot{\theta}_{ul}^i = -\frac{1}{2} \Gamma_{ul}^i e^T \mathbf{P} \mathbf{b}_c q_{ul}^i X, & \dot{\theta}_{ur}^i = -\frac{1}{2} \Gamma_{ur}^i e^T \mathbf{P} \mathbf{b}_c q_{ur}^i X, \\ \dot{\theta}_{ur}^j = -\frac{1}{2} \Gamma_{ur}^j e^T \mathbf{P} \mathbf{b}_c q_{ur}^j X, & \dot{\theta}_{ul}^j = -\frac{1}{2} \Gamma_{ul}^j e^T \mathbf{P} \mathbf{b}_c q_{ul}^j X. \end{cases} \quad (29)$$

所以

$$(\dot{\theta}_{ul}^k)^T = \begin{pmatrix} a_{ul0}^k \\ \vdots \\ a_{uln}^k \end{pmatrix} = -\frac{1}{2} e^T \mathbf{P} \mathbf{b}_c q_{ul}^i \begin{pmatrix} \tau_{ul0}^k \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad (\dot{\theta}_{ur}^k)^T = \begin{pmatrix} a_{ur0}^k \\ \vdots \\ a_{urn}^k \end{pmatrix} = -\frac{1}{2} e^T \mathbf{P} \mathbf{b}_c q_{ur}^i \begin{pmatrix} \tau_{ur0}^k \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

$k = 1, 2, \dots, M$, $(\dot{\sigma}_{ul}^k)^T$ 和 $(\dot{\sigma}_{ur}^k)^T$ 的表达类似.

则对生态位贴近度的参数求导, 并利用梯度下降法得到生态位贴近度的参数自适应率:

$$\begin{cases} \lambda_{ul}^k(\eta + 1) = -\frac{1}{2\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta) - \lambda)/(\sigma_{ul}^k(\eta) + \sigma)]^2/2} \frac{1}{\sigma_{ul}^k(\eta) + \sigma} \tau_{ul0}^k e^T \mathbf{P} \mathbf{b}_c q_{ul}^k, \\ \lambda_{ur}^k(\eta + 1) = -\frac{1}{2\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta) - \lambda)/(\sigma_{ur}^k(\eta) + \sigma)]^2/2} \frac{1}{\sigma_{ur}^k(\eta) + \sigma} \tau_{ur0}^k e^T \mathbf{P} \mathbf{b}_c q_{ur}^k, \\ \sigma_{ul}^k(\eta + 1) = -\frac{1}{2\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta) - \lambda)/(\sigma_{ul}^k(\eta) + \sigma)]^2/2} \frac{\lambda_{ul}^k(\eta) + \lambda}{(\sigma_{ul}^k(\eta) + \sigma)^2} \tau_{ul0}^k e^T \mathbf{P} \mathbf{b}_c q_{ul}^k, \\ \sigma_{ur}^k(\eta + 1) = -\frac{1}{2\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta) - \lambda)/(\sigma_{ur}^k(\eta) + \sigma)]^2/2} \frac{\lambda_{ur}^k(\eta) + \lambda}{(\sigma_{ur}^k(\eta) + \sigma)^2} \tau_{ur0}^k e^T \mathbf{P} \mathbf{b}_c q_{ur}^k, \end{cases} \quad (30)$$

$k = 1, 2, \dots, M.$

4 稳定性分析

定理 1 所有参数变量 $\|\theta_{ul}^k\| < M_u$, $\|\theta_{ur}^k\| < M_u$, $\|\sigma_{ur}^k\| < M_\sigma$, $\|\sigma_{ul}^k\| < M_\sigma$ 都有界, $k = 1, 2, \dots, M$.

证明 设 $V_{ul}^k = (\theta_{ul}^k)^T \theta_{ul}^k / 2$, 则

$$\dot{V}_{ul}^k = \frac{1}{2} (\dot{\theta}_{ul}^k)^T \theta_{ul}^k + \frac{1}{2} (\theta_{ul}^k)^T \dot{\theta}_{ul}^k.$$

如果

$$(\dot{\theta}_{ul}^k)^T = -\frac{1}{2} \Gamma_{ul}^i \mathbf{B}^T \mathbf{P} \tilde{e} q_{ul}^i X,$$

则当 $\|\theta_{ul}^k\| = M_u$ 时, $\|\dot{\theta}_{ul}^k\| \leq M_u$ 或 $\dot{V}_{ul}^k = -\frac{1}{2} \Gamma_{ul}^i \mathbf{B}^T \mathbf{P} \tilde{e} q_{ul}^i X \leq 0$ 成立. 因此, $\|\theta_{ul}^k\| < M_u$. 则

$\|\theta_{ul}^k\| = M_u$ 且

$$\dot{V}_{ul}^k = -\frac{1}{2} \Gamma_{ul}^i \mathbf{B}^T \mathbf{P} \tilde{e} q_{ul}^k \theta_{ul}^k \mathbf{X} + \frac{1}{2} \Gamma_{ul}^i \mathbf{B}^T \mathbf{P} \tilde{e} \frac{\theta_{ul}^k (\theta_{ul}^k)^T q_{ul}^k \theta_{ul}^k}{|\theta_{ul}^k|^2} = 0,$$

所以 $\|\theta_{ul}^k\| < M_u$. 同理可证

$$\|\theta_{ur}^k\| < M_u, \|\sigma_{ur}^k\| < M_\sigma, \|\sigma_{ul}^k\| < M_\sigma.$$

参数约束与收敛性分析可以详细参见文献[13-16], 这里就不再给予证明.

5 实例仿真

例 1 本例对肌型血管疾病^[18]进行控制仿真, 其数学模型为

$$\begin{cases} \frac{dx}{d\tau} = -mx - ny, \\ \frac{dy}{d\tau} = -(h + mh)x - (h + nh)y + hx^3 + E\cos(\omega\tau), \end{cases} \quad (31)$$

其中 x 是无量纲血管内径的变化差, y 是无量纲血管内的压力差, τ 是与时间成正比的“时间变量”, $E\cos(\omega\tau)$ 表示血管受到周期性刺激的干扰, m, n, h 为常参数.

在系统(31)第二式加上控制器 u , 则在控制输入的状态方程为

$$\begin{cases} \frac{dx}{d\tau} = -mx - ny, \\ \frac{dy}{d\tau} = -(h + mh)x - (h + nh)y + hx^3 + E\cos(\omega\tau) + u. \end{cases} \quad (32)$$

为了建立输出 y 与控制器 u 的直接关系, 需要对 y 求导, 令 $y = \dot{x}$, 经过求导后得

$$\begin{cases} \dot{y} = (h + mh)(mx + ny) - (h + mh)[-(h + mh)x - \\ \quad (h + nh)y + hx^3 + E\cos(\omega\tau) + u] + 3hx^2 - E\omega\sin(\omega\tau), \\ f(\mathbf{X}) = (h + mh)(mx + ny) - (h + mh)[-(h + mh)x - \\ \quad (h + nh)y + hx^3 + E\cos(\omega\tau)] + 3hx^2 - E\omega\sin(\omega\tau), \end{cases} \quad (33)$$

则式(33)可以写为

$$y = f(\mathbf{X}) + bu.$$

取 $m = -0.15, n = -1.7, h = -0.65, E = 0.3, \omega = 1$, 选择控制器参数 $k_1 = 2, k_2 = 1, \alpha = 10, \beta = 20, \theta_0 = 0$, 若取 $\mathbf{Q} = \text{diag}(2, 2)$, 则可求得 $d_0 = 1, d_1 = 1$. 选择 $u(0) = 0, \tau_{ur0}^k = 2, \tau_{ul0}^k = 2$, 取理想的 $\lambda = 0, \sigma = 1$, 则有自适应律:

$$\begin{cases} \lambda_{ul}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta)/(\sigma_{ul}^k(\eta)+1))^2]/2} \frac{1}{\sigma_{ul}^k(\eta) + 1} \mathbf{B}^T \mathbf{P} \tilde{e} q_{ul}^k, \\ \lambda_{ur}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta)/(\sigma_{ur}^k(\eta)+1))^2]/2} \frac{1}{\sigma_{ur}^k(\eta) + 1} \mathbf{B}^T \mathbf{P} \tilde{e} q_{ur}^k, \\ \sigma_{ul}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta)/(\sigma_{ul}^k(\eta)+1))^2]/2} \frac{\lambda_{ul}^k(\eta)}{(\sigma_{ul}^k(\eta) + 1)^2} \mathbf{B}^T \mathbf{P} \tilde{e} q_{ul}^k, \\ \sigma_{ur}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta)/(\sigma_{ur}^k(\eta)+1))^2]/2} \frac{\lambda_{ur}^k(\eta)}{(\sigma_{ur}^k(\eta) + 1)^2} \mathbf{B}^T \mathbf{P} \tilde{e} q_{ur}^k, \end{cases} \quad (34)$$

$$k = 1, 2, \dots, M.$$

令目标参考信号 $y_m(t) = 0$, 则仿真图如图 1~6 所示。

图 1 和图 2 表示系统及 2 个变量在此方法控制下稳定输出。从图 3 可以看出, 无论有无扰动, 本方法的系统输出曲线波动很小。图 4 表明观测器可以很好地跟踪系统的状态。图 5 表示对系统设计的自适应直接模糊控制器是有界的。图 6 反映了 type-2 控制比 type-1 控制效果更好。

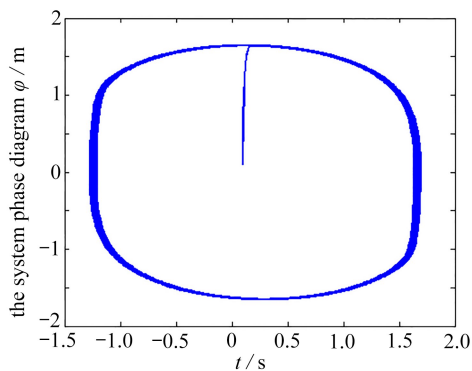


图 1 控制后系统相图

Fig. 1 The phase diagram after control

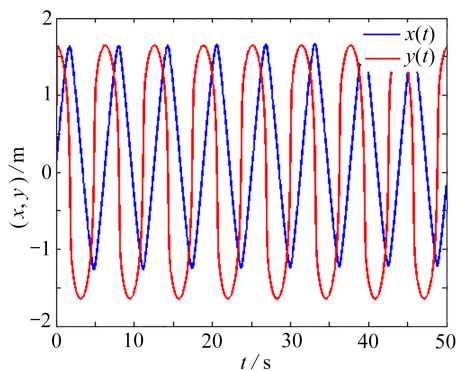


图 2 控制后各变量趋势图

Fig. 2 The trend chart of each variable after control

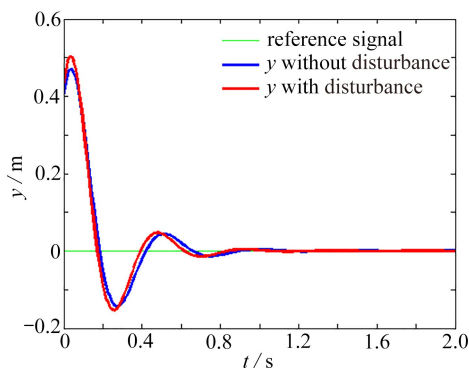


图 3 有无扰动的 y 输出比较

Fig. 3 Outputs with or without disturbance

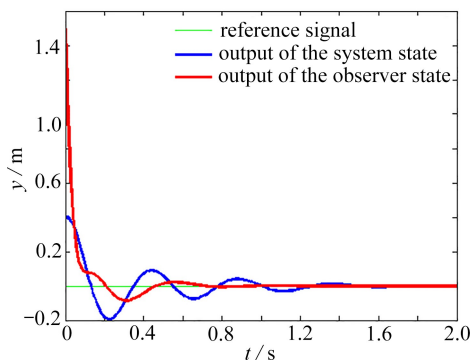


图 4 系统状态和直接状态系统比较

Fig. 4 The system state vs. the direct state system

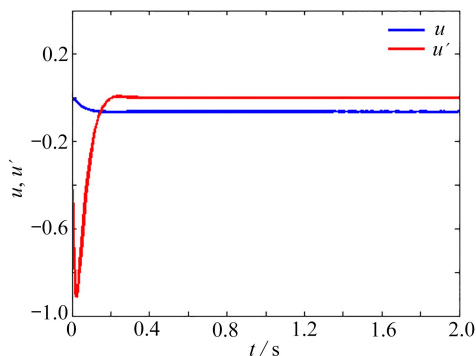


图 5 模糊控制器输出曲线

Fig. 5 Fuzzy controller output curves

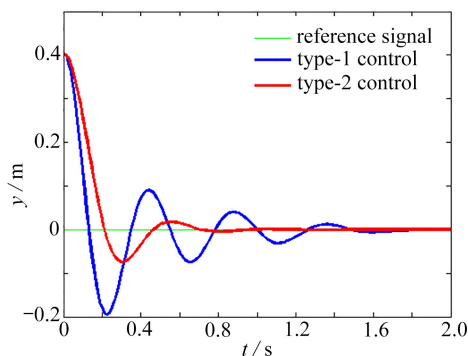


图 6 控制 type-1 型与 type-2 型比较

Fig. 6 Control comparison between type-1 and type-2

例 2 对 Willis 环状脑动脉瘤模型^[1]进行实例仿真, 其数学模型为

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -ax_2 - bx_1 + cx_1^2 - dx_1^3 + E\cos(\omega t), \end{cases} \quad (35)$$

其中 x_1 表示血流量, x_2 表示收缩血压, t 为时间, E 是冲量振幅, $\cos(\omega t)$ 表示中心血压的变化率, w 为心率的倒数, a, b, c, d 为常数. 当取 $a = 0.1, b = 1, c = 3, d = 2, E = 0.01, w = 1$. 系统出现混沌现象, 如图 7 所示.

为了控制系统, 在式 (35) 第二个方程加上控制器 u , 则在控制输入下的方程为

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -ax_2 - bx_1 + cx_1^2 - dx_1^3 + E\cos(\omega t) + u. \end{cases} \quad (36)$$

仿照上例计算方法, 令 $y = x_2$, 对 y 进行求导, 得到 $y = f(X) + bu$. 取理想的 $\lambda = 72.5, \sigma = 7.5$, $u(0) = 0$, 选取控制器参数 $k_1 = 1, k_2 = 1$, 则得自适应率:

$$\begin{cases} \lambda_{ul}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta) - 72.5)/(\sigma_{ul}^k(\eta) + 7.5)]^2/2} \frac{1}{\sigma_{ul}^k(\eta) + 1} \mathbf{B}^T \mathbf{P} \tilde{\mathbf{e}} q_{ul}^k, \\ \lambda_{ur}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta) - 72.5)/(\sigma_{ur}^k(\eta) + 7.5)]^2/2} \frac{1}{\sigma_{ur}^k(\eta) + 1} \mathbf{B}^T \mathbf{P} \tilde{\mathbf{e}} q_{ur}^k, \\ \sigma_{ul}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ul}^k(\eta) - 72.5)/(\sigma_{ul}^k(\eta) + 7.5)]^2/2} \frac{1}{(\sigma_{ul}^k(\eta) + 1)^2} \mathbf{B}^T \mathbf{P} \tilde{\mathbf{e}} q_{ul}^k, \\ \sigma_{ur}^k(\eta + 1) = -\frac{1}{\sqrt{2\pi}} e^{-[(\lambda_{ur}^k(\eta) - 72.5)/(\sigma_{ur}^k(\eta) + 7.5)]^2/2} \frac{\lambda_{ur}^k(\eta)}{(\sigma_{ur}^k(\eta) + 1)^2} \mathbf{B}^T \mathbf{P} \tilde{\mathbf{e}} q_{ur}^k, \end{cases} \quad (37)$$

$k = 1, 2, \dots, M.$

仿真中, 取 $x_1(0) = 0.5, x_2(0) = 1, y_i = 0$, 则仿真图如图 7~10 所示.

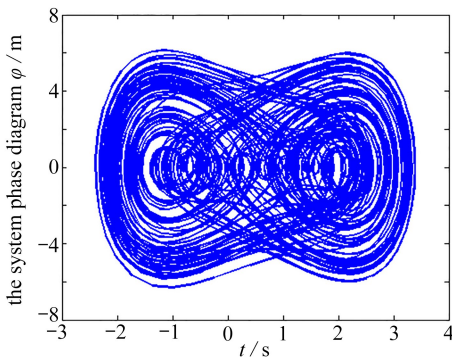


图 7 系统控制前混沌状态

Fig. 7 The chaotic state before system control

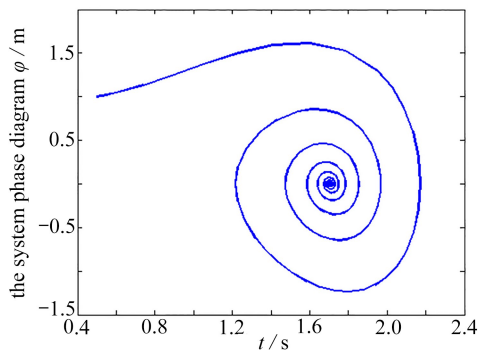


图 8 控制后的系统相图

Fig. 8 The system phase diagram after control

图 7 反映系统在控制之前成混沌状态, 经本方法控制后, 系统成稳定状态 (图 8). 图 9 表示系统 2 个变量在此方法控制下稳定输出. 从图 10 可以看出, type-2 控制比 type-1 控制的波动较小, 其控制收敛效果更好.

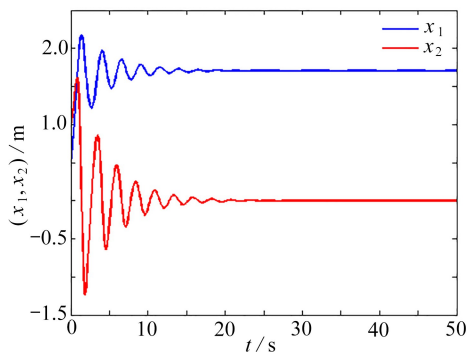


图9 控制后的变量输出图

Fig. 9 The variable output curves after control

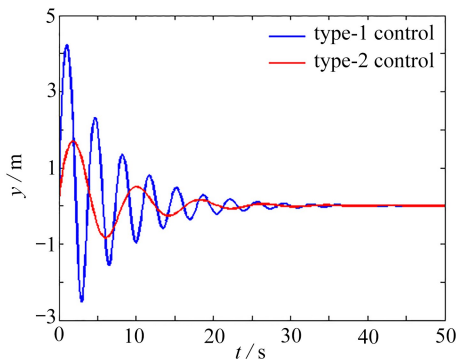


图10 控制 type-1 型与 type-2 型比较

Fig. 10 Control comparison between type-1 and type-2

6 结 论

近年来生态环境的破坏制约着经济的发展、影响着人们的身心健康,生态位在生态系统中起着重要的作用,生物个体有朝着追求最佳生态位的趋向.生态位的模糊建模很好地刻画了生物个体在环境中的定位, type-2 模糊控制在稳定一类具有参数不确定性的非线性系统时,具有不错的鲁棒性.将系统生物个体的进化特性、自适应性和稳定性与 type-2 直接 T-S 模糊自适应控制方法相结合,生态位的贴进度函数作为 T-S 模糊控制的后件,构造具有生物特性的 T-S 模糊控制方法,求得生态因子的自适应率,反映了生物个体的自适应.并通过实例对比本控制方法的有效性,从仿真图可以看到,此控制方法在收敛速度上更快,晃动幅度更小,进一步反映了本控制方法的优越性.也有利于环境的和谐和生态系统的稳定及生态环境的可持续发展.

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Type-2 Direct T-S Fuzzy Control of Niche Equality Indexes

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Abstract: Given the important role of niches in the ecosystem and the operability of type-2 direct T-S fuzzy control of the stability of a class of nonlinear systems with parameter uncertainties, the biological individuals' evolutionary characteristics and adaptive behaviors were integrated with the direct T-S fuzzy type-2 control method, and the niche closeness function was used as a follow-up to type-2 T-S fuzzy control parts. Besides, a type-2 direct T-S fuzzy control method with biological characteristics was proposed to find the niche ecology. The self-adaptation rate of the factor reflects the degree of the adaptive use of the environment by biological individuals. Through comparison of simulation examples, this study reveals that type-2 is superior to type-1 in terms of stability and convergence. The study shows that the type-2 method is conducive to environmental harmony, ecological stability and sustainable development of ecological environment; in the meantime, this method gives fuzzy control a practical physical background.

Key words: niche; type-2; fuzzy control; equality index; sustainable development

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