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基于 Lévy 噪声的混合时滞中立型神经网络自适应同步研究*

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摘要: 研究了带有 Lévy 噪声的混合时滞随机中立型神经网络的自适应同步问题.Lévy 噪声的提出,使得网络里的噪声干扰由 Gauss 过程和 Poisson 点过程两部分组成,同时包含了连续的扰动和不连续的突触噪声.通过建立新的 Lyapunov 泛函,使用 Itôs 公式以及不等式分析方法,得到误差系统的稳定性条件,给出了反馈控制器的更新率,从而进一步保证响应系统和驱动系统的自适应同步.最后,提供了一个数值实例,通过 MATLAB 相关仿真,说明前文所得结果的正确性.

关 键 词: 中立型神经网络; Lévy 噪声; 自适应同步; 混合时滞

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引 言

在神经网络中,时滞延迟是不可避免的问题,它可能会破坏网络的稳定性,甚至会出现混沌现象.因此,时滞系统的动力学行为分析具有重要现实价值.中立型神经网络是一类特殊的时滞系统,其模型结构特点是不仅系统状态中存在时滞,而且系统状态的导数中也存在时滞,即系统状态的演化依赖于现在的状态和过去时刻状态的变化率.由于中立型神经网络的特殊性,激起了很多学者的研究兴趣(见文献[1-3]).但中立型神经网络的系统模型更加复杂,已经取得的很多结果不能简单地套用,所以有关中立型神经网络的研究成果有待进一步完善.

众所周知,稳定和同步问题是神经网络研究中非常重要的问题.对于含有不同时变时滞的神经网络系统,得到了很多研究者的关注,已有文献研究了很多有关稳定性的结果(见文献[4-7]).其中文献[4]研究了基于加性时滞和混合时滞的中立型神经网络的稳定性;文献[5]研究了具有连续时滞的神经网络的渐近稳定性.此外,对于神经网络同步开始广泛用在信号处理、组合优化和信息安全等领域.现在已经存在很多有关神经网络同步的研究(见文献[8-14]).比如文献[11]讨论了两个混沌神经网络的自适应同步问题;文献[12]研究了有关模糊控制系统的自适应控制问题.

在有关研究中立型神经网络稳定性和同步性的文献中,Gauss 白噪声或 Brown 运动被认

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为是描述神经网络中出现扰动的一种常用模型.然而 Gauss 白噪声是连续的运动轨迹,不足以描述由突触传递所产生的不连续突触噪声.幸运的是,Poisson 点过程可以用来描述这种突触噪声.研究者们为了更好地对系统噪声进行建模,提出了 Lévy 噪声,它由 Gauss 过程和 Poisson 点过程两部分组成,包含了连续的 Gauss 白噪声和不连续的突触噪声,这样使得网络系统中的噪声能够更加全面且贴合实际.关于具有 Lévy 噪声的随机微分系统的稳定性研究,见文献[15-17].文献[9]研究了具有 Gauss 噪声的时滞神经网络,并没有使用 Lévy 噪声;文献[16]研究了具有 Lévy 噪声的时滞网络稳定性,模型中的时滞并没有同时依赖于离散和分布时滞.本文在中立型神经网络系统中的扰动使用的是 Lévy 噪声,时滞为包含离散和分布两部分的混合时滞,考虑了更多的时滞信息,所得的结论相对具有较小的保守性.到目前为止,基于 Lévy 噪声的混合时滞中立型神经网络的自适应同步问题还没有引起足够的重视.

本文主要研究基于 Lévy 噪声的混合时滞中立型神经网络的 p 阶自适应指数同步,所采用的方法包括线性矩阵不等式方法、Lyapunov 泛函方法和不等式分析技术,由此得到网络的 p 阶自适应指数同步准则和控制器的更新率.

主要创新点为以下三点:

1) 建立了基于 Lévy 噪声的混合时滞中立型神经网络模型;

2) 本文提出的 Lyapunov 泛函可以解决同时存在混合时滞中立型和 Lévy 噪声的问题;

3) 设计了新的控制器更新率,得到响应系统和驱动系统是 p 阶自适应指数同步准则.

1　问题描述

神经网络动力学经常被建模为带 Gauss 白噪声的随机系统,而本文网络模型中的扰动是将 Gauss 白噪声推广到了 Lévy 噪声,且在中立型神经网络中添加了包含离散和分布两部分的混合时滞,所得的结论相对具有较小的保守性.

考虑如下带有 Lévy 噪声的混合时滞随机中立型神经网络模型:

$$
\begin{aligned}
&\mathrm{d}[\boldsymbol{x}(t)-\boldsymbol{D}^{r(t)}\boldsymbol{x}(t-h_0)]=\\
&\quad\Big[-\boldsymbol{C}^{r(t)}\boldsymbol{x}(t)+\boldsymbol{A}_0^{r(t)}\boldsymbol{f}_0(\boldsymbol{x}(t))+\sum_{q=1}^{m}\boldsymbol{A}_q^{r(t)}\boldsymbol{f}_q(\boldsymbol{x}(t-h_q(t)))+\\
&\quad\boldsymbol{B}^{r(t)}\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s\Big]\mathrm{d}t+\\
&\quad\boldsymbol{g}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))\mathrm{d}B(t)+\\
&\quad\int_{\mathbf{R}}\boldsymbol{h}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))N(\mathrm{d}t,\mathrm{d}z),
\end{aligned}
\tag{1}
$$

其中 $\boldsymbol{x}(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^{\mathrm{T}}\in R^n$ 表示 n 个神经元的状态向量, $\boldsymbol{f}(\cdot)$ 是神经元激活函数, $h_q(t),\delta(t)$ 是传递时滞.对于 $t\geqslant 0,\boldsymbol{C}^{r(t)}=\mathrm{diag}\{c_1^{r(t)},c_2^{r(t)},\cdots,c_n^{r(t)}\}$ 是正定对角矩阵, $\boldsymbol{A}_0^{r(t)},\boldsymbol{A}_q^{r(t)},\boldsymbol{B}^{r(t)},\boldsymbol{D}^{r(t)}$ 分别表示连接权矩阵和时滞连接权矩阵, $B(t)$ 是定义在完备概率空间的一维标准 Brown 运动, $N(\mathrm{d}t,\mathrm{d}z)$ 表示一个一维 F_t- 适应的 Poisson 随机测度. $\boldsymbol{g}:\mathbf{R}_+\times S\times R^n\times\cdots\times R^n\to R^{n\times m}$ 和 $\boldsymbol{h}:\mathbf{R}_+\times S\times\mathbf{R}\times R^n\times\cdots\times R^n\to R^n$ 分别是连续噪声强度矩阵和不连续噪声强度矩阵.

令 $\{r(t),t\geqslant 0\}$ 是一个取值于完备概率空间 $S=\{1,2,\cdots,S\}$ 上的右连续 Markov 链,其转移矩阵 $\boldsymbol{\Gamma}=(\gamma_{ij})_{S\times S}$ 表示成

$$
P\{r(t+\delta)=j\mid r(t)=i\}=\begin{cases}\gamma_{ij}\delta+o(\delta), & i\neq j,\\ 1+\gamma_{ii}\delta+o(\delta), & i=j,\end{cases}
$$

其中$\delta > 0, \gamma_{ij} \geqslant 0$是$i \neq j$时从$i$到$j$的传输速率, $\gamma_{ii} = -\sum_{j=1, j\neq i}^{S} \gamma_{ij}$.

主系统(1)所对应的从系统如下:

$$\begin{aligned}
&\mathrm{d}[\boldsymbol{y}(t) - \boldsymbol{D}^{r(t)}\boldsymbol{y}(t-h_0)] = \\
&\quad\Big[-\hat{\boldsymbol{C}}^{r(t)}\boldsymbol{y}(t) + \hat{\boldsymbol{A}}_0^{r(t)}\boldsymbol{f}_0(\boldsymbol{y}(t)) + \sum_{q=1}^{m}\hat{\boldsymbol{A}}_q^{r(t)}\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t))) + \\
&\quad\hat{\boldsymbol{B}}^{r(t)}\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s + \boldsymbol{U}(t,r(t))\Big]\mathrm{d}t + \\
&\quad\boldsymbol{g}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t)))\mathrm{d}B(t) + \\
&\quad\int_{\mathbf{R}}\boldsymbol{h}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t)))N(\mathrm{d}t,\mathrm{d}z),
\end{aligned} \tag{2}$$

其中$\boldsymbol{y}(t)$是网络的状态向量, $\hat{\boldsymbol{C}}^{r(t)},\hat{\boldsymbol{A}}_0^{r(t)},\hat{\boldsymbol{A}}_q^{r(t)},\hat{\boldsymbol{B}}^{r(t)}$是对应于$\boldsymbol{C}^{r(t)},\boldsymbol{A}_0^{r(t)},\boldsymbol{A}_q^{r(t)},\boldsymbol{B}^{r(t)}$的估计量, $\boldsymbol{U}(t,r(t))$是同步控制器.

令$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t)$, 记$r(t) = i$, 由式(1)和(2)易得

$$\begin{aligned}
&\mathrm{d}[\boldsymbol{e}(t) - \boldsymbol{D}^{r(t)}\boldsymbol{e}(t-h_0)] = \\
&\quad\Big[-\tilde{\boldsymbol{C}}^{i}\boldsymbol{y}(t) - \boldsymbol{C}^{i}\boldsymbol{e}(t) + \tilde{\boldsymbol{A}}_0^{i}\boldsymbol{f}_0(\boldsymbol{y}(t)) + \boldsymbol{A}_0^{i}\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t)) + \\
&\quad\sum_{q=1}^{m}\tilde{\boldsymbol{A}}_q^{i}\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t))) + \sum_{q=1}^{m}\boldsymbol{A}_q^{i}\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q)) + \\
&\quad\tilde{\boldsymbol{B}}^{i}\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s + \boldsymbol{B}^{i}\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s + \boldsymbol{U}(t,r(t))\Big]\mathrm{d}t + \\
&\quad[\boldsymbol{g}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t))) - \\
&\quad\boldsymbol{g}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))]\mathrm{d}B(t) + \\
&\quad\int_{\mathbf{R}}[\boldsymbol{h}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t))) - \\
&\quad\boldsymbol{h}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))]N(\mathrm{d}t,\mathrm{d}z),
\end{aligned} \tag{3}$$

其中

$$\begin{aligned}
&\tilde{\boldsymbol{C}}^{i} = \hat{\boldsymbol{C}}^{r(t)} - \boldsymbol{C}^{r(t)},\ \tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t)) = \boldsymbol{f}_0(\boldsymbol{y}(t)) - \boldsymbol{f}_0(\boldsymbol{x}(t)),\\
&\tilde{\boldsymbol{A}}_0^{i} = \hat{\boldsymbol{A}}_0^{r(t)} - \boldsymbol{A}_0^{r(t)},\ \tilde{\boldsymbol{A}}_q^{i} = \hat{\boldsymbol{A}}_q^{r(t)} - \boldsymbol{A}_q^{r(t)},\ \tilde{\boldsymbol{B}}^{i} = \hat{\boldsymbol{B}}^{r(t)} - \boldsymbol{B}^{r(t)}.
\end{aligned}$$

系统(3)的初始条件为$\boldsymbol{e}(s) = \boldsymbol{\xi}_{\boldsymbol{e}}(s), s \in [-\bar{h}, 0]$, 令

$$\begin{aligned}
&\boldsymbol{F}(t) = \boldsymbol{F}(t,\boldsymbol{x}(t),\boldsymbol{y}(t),r(t),\boldsymbol{e}(t),\boldsymbol{e}_{h_1}(t),\cdots,\boldsymbol{e}_{h_m}(t)) = \\
&\quad -\tilde{\boldsymbol{C}}^{i}\boldsymbol{y}(t) - \boldsymbol{C}^{i}\boldsymbol{e}(t) + \tilde{\boldsymbol{A}}_0^{i}\boldsymbol{f}_0(\boldsymbol{y}(t)) + \boldsymbol{A}_0^{i}\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t)) + \\
&\quad\sum_{q=1}^{m}\tilde{\boldsymbol{A}}_q^{i}\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t))) + \sum_{q=1}^{m}\boldsymbol{A}_\theta^{i}\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q)) + \\
&\quad\tilde{\boldsymbol{B}}^{i}\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s + \boldsymbol{B}^{i}\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s + \boldsymbol{U}(t,r(t)),\\
&\boldsymbol{G}(t) = \boldsymbol{G}(t,\boldsymbol{x}(t),\boldsymbol{y}(t),r(t),\boldsymbol{e}(t),\boldsymbol{e}_{h_1}(t),\cdots,\boldsymbol{e}_{h_m}(t)) = \\
&\quad\boldsymbol{g}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t))) - \\
&\quad\boldsymbol{g}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t))),\\
&\boldsymbol{H}(t,z) = \boldsymbol{H}(t,\boldsymbol{x}(t),\boldsymbol{y}(t),r(t),\boldsymbol{e}(t),\boldsymbol{e}_{h_1}(t),\cdots,\boldsymbol{e}_{h_m}(t)) = \\
&\quad\boldsymbol{h}(t,r(t),\boldsymbol{y}(t),\boldsymbol{y}(t-h_1(t)),\cdots,\boldsymbol{y}(t-h_m(t))) - \\
&\quad\boldsymbol{h}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t))),
\end{aligned}$$

则式(3)可以简记为

$$\mathrm{d}[\boldsymbol{e}(t)-\boldsymbol{D}^{r(t)}\boldsymbol{e}(t-h_0)]=\boldsymbol{F}(t)\mathrm{d}t+\boldsymbol{G}(t)\mathrm{d}B(t)+\int_{R^n}\boldsymbol{H}(t,z)N(\mathrm{d}t,\mathrm{d}z). \tag{4}$$

时滞状态反馈控制器如下：

$$\boldsymbol{U}(i,t)=\boldsymbol{K}^i[\boldsymbol{e}(t)+\boldsymbol{\psi}(\boldsymbol{e}(t))],\ \boldsymbol{\psi}(\boldsymbol{e}(t))=\boldsymbol{\varphi}(\boldsymbol{y}(t))-\boldsymbol{\varphi}(\boldsymbol{x}(t)). \tag{5}$$

假设 1 对于中立项参数矩阵 $\boldsymbol{D}^i(i=1,2,\cdots,S)$，存在 $\kappa_i\in(0,1)$，κ_i 为正常数，使得 $\rho(\boldsymbol{D}^i)=\kappa_i\leqslant\kappa\in(0,1)$，其中 $\kappa=\max_{i\in S}\kappa_i$，且 $\rho(\boldsymbol{D}^i)$ 是矩阵 $\boldsymbol{D}^i$ 的谱半径.

假设 2 对于激活函数 $\boldsymbol{f}(\cdot)$ 满足 Lipschitz 条件，存在常数 $L>0$，使得

$$|\boldsymbol{f}(\boldsymbol{x})-\boldsymbol{f}(\boldsymbol{y})|\leqslant L|\boldsymbol{x}-\boldsymbol{y}|,\qquad\forall\boldsymbol{x},\boldsymbol{y}\in R^n.$$

假设 3 特征测度 $\upsilon(\mathrm{d}z)\mathrm{d}t$ 满足 $\upsilon(\mathrm{d}z)\mathrm{d}t=\lambda\mu(\mathrm{d}z)\mathrm{d}t$，使得 Poisson 点过程是平稳过程，其中 λ 是 Poisson 分布的强度，而 μ 是随机变量 z 的概率分布.

假设 4 连续噪声强度矩阵 $\boldsymbol{g}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))$ 满足 Lipschitz 条件，存在正常数 $G,G_q,G_\delta,\bar{\eta},\eta_0,\eta_q,\eta_\delta,q=1,2,\cdots,m$ 使得

$$\begin{aligned}&\mathrm{trace}[\boldsymbol{G}^{\mathrm{T}}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))\cdot\\&\quad\boldsymbol{G}(t,r(t),\boldsymbol{x}(t),\boldsymbol{x}(t-h_1(t)),\cdots,\boldsymbol{x}(t-h_m(t)))]\leqslant\\&\quad\boldsymbol{G}|\boldsymbol{e}(t)|^2+\boldsymbol{G}_1|\boldsymbol{e}_{h_1}(t)|^2+\cdots+\boldsymbol{G}_q|\boldsymbol{e}_{h_q}(t)|^2+\boldsymbol{G}_\delta|\boldsymbol{e}_\delta(t)|^2,\end{aligned}$$

$$\begin{aligned}&\int_{\mathbf{R}}[|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}+\boldsymbol{H}(t,\boldsymbol{x}(t),\boldsymbol{y}(t),r(t),\boldsymbol{e}(t),\boldsymbol{e}_{h_1}(t),\cdots,\boldsymbol{e}_{h_m}(t))|^p-\\&\quad|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p]\nu\mathrm{d}z\leqslant\\&\quad\bar{\eta}|\boldsymbol{e}(t)|^p+\eta_0|\boldsymbol{e}_{h_0}(t)|^p+\eta_1|\boldsymbol{e}_{h_1}(t)|^p+\cdots+\eta_q|\boldsymbol{e}_{h_q}(t)|^p+\eta_\delta|\boldsymbol{e}_\delta(t)|^p.\end{aligned}$$

下面我们给出系统 p 阶自适应指数同步的定义如下.

定义 1 对 $\forall\boldsymbol{\zeta}(\theta)\in L^p_{F_0}([-\bar{\tau},0],R^n)$，使得

$$\limsup_{t\to\infty}\frac{1}{t}\lg(\varepsilon|\boldsymbol{e}(t,\boldsymbol{\zeta}(\theta))|^p)<0,$$

则误差系统(3)的平凡解 $\boldsymbol{e}(t,\boldsymbol{\zeta}(\theta))$ 是 p 阶指数稳定的.当误差系统(3)是 p 阶指数稳定的，主系统(1)和从系统(2)是 p 阶指数同步的.

定义 2 如果一个随机过程 $\{\boldsymbol{x}(t)|t\in[0,+\infty)\}$ 满足下面的条件，则它是一个 Lévy 过程：

1) $\boldsymbol{x}(0)=\mathbf{0}$；

2) $\boldsymbol{x}(t)$ 有独立平稳增量；

3) 对于 $\forall a>0,\forall s\geqslant0,\boldsymbol{x}(t)$ 是随机连续的.

引理 1[18] 假设 $\boldsymbol{x}\in R^n$，$\boldsymbol{y}\in R^n$，那么有

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{x}\leqslant\varpi\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+\varpi^{-1}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}$$

对 $\forall\varpi>0$ 都成立.

引理 2(Young 不等式[19]) 假设 $a,b\in\mathbf{R}$ 且 $\beta\in[0,1]$，那么

$$|a|^\beta|b|^{1-\beta}\leqslant\beta|a|+(1-\beta)|b|.$$

引理 3[20] 假设 $p>1$，存在常数 $\mu\in(0,1)$，使得 $|\boldsymbol{D}(\boldsymbol{z},i)|\leqslant\mu|\boldsymbol{z}|$，则

$$|\boldsymbol{x}-\boldsymbol{D}(\boldsymbol{z},i)|^p\leqslant(1+\mu)^{p-1}(|\boldsymbol{x}|^p+\mu|\boldsymbol{z}|^p),\qquad\forall(\boldsymbol{x},\boldsymbol{z},i)\in R^N\times R^N\times S. \tag{6}$$

引理 4[20] 假设 $p>1$，存在一个常数 $\mu\in(0,1)$，使得 $|\boldsymbol{D}(\boldsymbol{z},i)|\leqslant\mu|\boldsymbol{z}|$，那么有

$$|\boldsymbol{x}|^p\leqslant\mu|\boldsymbol{z}|^p+\frac{|\boldsymbol{x}-\boldsymbol{D}(\boldsymbol{z},i)|^p}{(1-\mu)^{p-1}},\qquad\forall(\boldsymbol{x},\boldsymbol{z},i)\in R^N\times R^N\times S. \tag{7}$$

使用 Hölder 不等式,得到

$$-|\boldsymbol{x}-\boldsymbol{D}(z,i)|^{p} \leqslant -(1-\mu)^{p-1}|\boldsymbol{x}|^{p}+\mu(1-\mu)^{p-1}|\boldsymbol{z}|^{p}. \tag{8}$$

引理 5[21] 存在正定对称矩阵 $\boldsymbol{R}$ 和任意可微函数 $\boldsymbol{w}:[a,b]\to R^{n}$, 那么有

$$\int_{a}^{b}\boldsymbol{w}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{w}(s)\mathrm{d}s \geqslant \frac{1}{b-a}\left(\int_{a}^{b}\boldsymbol{w}(s)\mathrm{d}s\right)^{\mathrm{T}}\boldsymbol{R}\left(\int_{a}^{b}\boldsymbol{w}(s)\mathrm{d}s\right). \tag{9}$$

2 主要结果

给定 $\boldsymbol{V}(\boldsymbol{x},t,i) \in C^{2,1}(\mathbf{R}_{+}\times S\times R^{n}\times\cdots\times R^{n};\mathbf{R}_{+})$, 定义算子

$$\begin{aligned}
&L\boldsymbol{V}(t,i,\boldsymbol{e},\boldsymbol{e}_{h_0})=\\
&\quad \boldsymbol{V}_{t}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0})+\boldsymbol{V}_{\boldsymbol{x}}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0})\boldsymbol{F}(t)+\\
&\quad \frac{1}{2}\mathrm{trace}(\boldsymbol{G}^{\mathrm{T}}(t)\boldsymbol{V}_{\boldsymbol{xx}}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0})\boldsymbol{G}(t))+\\
&\quad \int_{\mathbf{R}}[\boldsymbol{V}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0}+\boldsymbol{H}(t,z))-\boldsymbol{V}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0})]\nu\mathrm{d}z+\\
&\quad \sum_{j=1}^{N}\gamma_{ij}\boldsymbol{V}(t,j,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0}),
\end{aligned} \tag{10}$$

其中

$$\boldsymbol{V}_{t}(t,i,\boldsymbol{x})=\frac{\partial\boldsymbol{V}(t,i,\boldsymbol{e}-\boldsymbol{D}^{i}\boldsymbol{e}_{h_0})}{\partial t},\ \boldsymbol{V}_{\boldsymbol{x}}(t,i,\boldsymbol{x})=\left(\frac{\partial\boldsymbol{V}(t,i,\boldsymbol{x})}{\partial x_1},\cdots,\frac{\partial\boldsymbol{V}(t,i,\boldsymbol{x})}{\partial x_n}\right),$$

$$\boldsymbol{V}_{\boldsymbol{xx}}(t,i,\boldsymbol{x})=\left(\frac{\partial^{2}\boldsymbol{V}(t,i,\boldsymbol{x})}{\partial x_j\partial x_j}\right)_{n\times n}.$$

定理 1 假设存在函数 $\boldsymbol{V}(\boldsymbol{e},t,i) \in C^{2,1}(R^{n}\times S\times\mathbf{R}_{+};\mathbf{R}_{+})$ 及 4 个正常数 $p,\mu_1,\lambda_1,\lambda_2$ 使得

$$\lambda_2<\lambda_1,\ \mu_1|\boldsymbol{e}|^{p}\leqslant\boldsymbol{V}(\boldsymbol{e},t,i), \tag{11}$$

$$L\boldsymbol{V}(t,\boldsymbol{e},\boldsymbol{e}_h,i)\leqslant-\lambda_1|\boldsymbol{e}|^{p}+\lambda_2|\boldsymbol{e}_h|^{p},\qquad \forall t\geqslant 0,\ i\in S,\ \boldsymbol{e}\in R^{n}, \tag{12}$$

$$\lim_{t\to\infty}\sup\frac{1}{t}\lg(E|\boldsymbol{e}(t,\boldsymbol{\xi}(s))|^{p})<0,\qquad \forall\boldsymbol{\xi}(s)\in L_{F_0}^{p}([-\bar{h},0];R^{n}), \tag{13}$$

那么,系统是 p 阶指数稳定的.

定理 2 假设

$$\bar{\iota}+\frac{1}{1-\hat{h}}\sum_{q=0}^{m}\iota_q<0, \tag{14}$$

其中

$$\kappa=H_1+H_2+H_3+H_4-a_1(1-\mu)^{p-1}+a_2(1+\mu)^{p-1},$$

$$\bar{\iota}=w_i\kappa+(1+\mu)^{p-1}\sum_{j=1}^{N}\gamma_{ij}q_j+\bar{\eta},$$

$$\iota_0=(J_1+J_2+J_3+J_4)w_i+a_2c_2\mu(1-\mu)^{p-1}+b_1\mu+\eta_0,\ \iota_q=L^{2}+\eta_q+G_q,$$

且

$$\begin{cases}\delta S-(1-\mu)Q\leqslant 0,\\ \phi=\min\limits_{i\in S}\min\limits_{1\leqslant j\leqslant n}|\boldsymbol{C}_j^{i}|,\ \alpha=\max\limits_{i\in S}(\rho(\boldsymbol{A}_0^{i}))^{2},\\ \beta=\max\limits_{i\in S}(\rho(\boldsymbol{A}_q^{i}))^{2},\chi=\max\limits_{i\in S}(\rho(\boldsymbol{B}^{i}))^{2}.\end{cases} \tag{15}$$

反馈控制器 $\boldsymbol{U}(i,t)$ 的更新率选择为

$$\dot{\boldsymbol{k}}_j^i=-\frac{1}{2}\zeta_j pw_i|\boldsymbol{e}_j(t)-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}[\boldsymbol{e}_j(t)-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j]^2, \tag{16}$$

且矩阵 $\hat{\boldsymbol{C}}^i,\hat{\boldsymbol{A}}_0^i,\hat{\boldsymbol{A}}_q^i,\hat{\boldsymbol{B}}^i$ 的参数更新率选择如下：

$$\begin{cases}\dot{\tilde{\boldsymbol{c}}}_j^i=\dfrac{\phi_j}{2}pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}[\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j]^2\boldsymbol{y}_j,\\ \dot{\tilde{\boldsymbol{a}}}_{0jk}^i=-\dfrac{\alpha_{jk}}{2}pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}[\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j]^2[\boldsymbol{f}_0(\boldsymbol{y}(t))]_k,\\ \dot{\tilde{\boldsymbol{a}}}_{qjk}^i=-\dfrac{\beta_{jk}}{2}pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}[\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j]^2[\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t)))],\\ \dot{\tilde{\boldsymbol{b}}}_{jk}^i=\dfrac{\chi_{jk}}{2}pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0))^{\mathrm{T}}\displaystyle\int_{t-\delta}^t\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s,\end{cases} \tag{17}$$

其中 $\zeta_j>0,\phi_j>0,\alpha_{jk}>0,\beta_{jk}>0,\chi_{jk}>0(j,k=1,2,\cdots,n)$ 分别是任意的正常数.

那么,响应系统(2)和驱动系统(1)是 p 阶自适应指数同步的.

证明 选择如下的 Lyapunov 函数:

$$\boldsymbol{V}(t,r(t),\boldsymbol{x}(t))=\boldsymbol{V}_1+\boldsymbol{V}_2+\boldsymbol{V}_3+\boldsymbol{V}_4+\boldsymbol{V}_5+\boldsymbol{V}_6,$$

其中

$$\boldsymbol{V}_1=w_i|\boldsymbol{x}(t)|^p,\ \boldsymbol{V}_2=\sum_{j=1}^n\frac{1}{\xi_j}\boldsymbol{k}_j^2,\ \boldsymbol{V}_3=\sum_{j=1}^n\frac{1}{\phi_j}(\tilde{\boldsymbol{c}}_j^i)^2,$$

$$\boldsymbol{V}_4=\sum_{j=1}^n\sum_{k=1}^n\frac{1}{\alpha_{jk}}(\tilde{\boldsymbol{a}}_{0jk}^i)^2,\ \boldsymbol{V}_5=\sum_{j=1}^n\sum_{q=1}^n\sum_{k=1}^n\frac{1}{\beta_{jk}}(\tilde{\boldsymbol{a}}_{qjk}^i)^2,$$

$$\boldsymbol{V}_6=\int_{t-\delta(t)}^t\int_\theta^t\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{Q}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\mathrm{d}\theta+\sum_{j=1}^n\sum_{k=1}^n\frac{1}{\chi_{jk}}(\tilde{\boldsymbol{b}}_{jk}^i)^2.$$

令 $\boldsymbol{X}=\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0)$, 由式(12)和(13)计算算子 $L\boldsymbol{V}_1$ 可得

$$\begin{aligned}L\boldsymbol{V}_1={}&pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0))^{\mathrm{T}}\Big\{-\boldsymbol{C}^i\boldsymbol{y}(t)-\boldsymbol{C}^i\boldsymbol{e}(t)+\\ &\tilde{\boldsymbol{A}}_0^i\boldsymbol{f}_0(y)+\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))+\sum_{q=1}^m\tilde{\boldsymbol{A}}_q^i\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t)))+\sum_{q=1}^m\boldsymbol{A}_q^i\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q))+\\ &\tilde{\boldsymbol{B}}^i\int_{t-\delta(t)}^t\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s+\boldsymbol{B}^i\int_{t-\delta(t)}^t\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s+\boldsymbol{U}(t,r(t))\Big\}+\\ &\frac{1}{2}\mathrm{trace}[\boldsymbol{G}^{\mathrm{T}}(p(p-2)w_i|\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0)|^{p-4}((\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0))^{\mathrm{T}})^2+\\ &pw_i|\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0)|^{p-2})\boldsymbol{G}]+\sum_{j=1}^N\gamma_{ij}w_j|\boldsymbol{x}(t)|^p+\\ &\int_{\mathbf{R}}(w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}+\boldsymbol{H}|^p-w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p)\nu\mathrm{d}z\leqslant\\ &pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0))^{\mathrm{T}}\Big\{-\boldsymbol{C}^i\boldsymbol{y}(t)-\boldsymbol{C}^i\boldsymbol{e}(t)+\\ &\tilde{\boldsymbol{A}}_0^i\boldsymbol{f}_0(y)+\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))+\sum_{q=1}^m\tilde{\boldsymbol{A}}_q^i\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t)))+\sum_{q=1}^m\boldsymbol{A}_q^i\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q))\Big\}+\end{aligned}$$

$$\frac{1}{2}\operatorname{trace}[\boldsymbol{G}^{\mathrm{T}}(p(p-1)w_i|\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}(t-h_0)|^{p-2})\boldsymbol{G}]+\sum_{j=1}^{N}\gamma_{ij}w_j|\boldsymbol{x}(t)|^p+$$

$$\int_{\mathbf{R}}(w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}+\boldsymbol{H}|^p-w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p)\nu\mathrm{d}z. \tag{18}$$

利用式(6)分别计算以下算子:

$$LV_2=2\sum_{j=1}^{n}\frac{1}{\xi_j}\boldsymbol{k}_j\dot{\boldsymbol{k}}_j=-\sum_{j=1}^{n}\boldsymbol{k}_jpw_i(\boldsymbol{e}_j(t)-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j(t)-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}=$$

$$-kpw_i(\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}|\boldsymbol{e}(t)-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}, \tag{19}$$

$$LV_3=2\sum_{j=1}^{n}\frac{1}{\phi_j}\tilde{\boldsymbol{c}}_j^i\dot{\tilde{\boldsymbol{c}}}_j^i=\sum_{j=1}^{n}pw_i\tilde{\boldsymbol{c}}_j^i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}\boldsymbol{y}_j(t)=$$

$$pw_i(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}\tilde{\boldsymbol{C}}^i\boldsymbol{y}(t), \tag{20}$$

$$LV_4=2\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{1}{\alpha_{jk}}\tilde{\boldsymbol{a}}_{0jk}^i\dot{\tilde{\boldsymbol{a}}}_{0jk}^i=$$

$$-\sum_{j=1}^{n}\sum_{k=1}^{n}pw_i\tilde{\boldsymbol{a}}_{0jk}^i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}[\boldsymbol{f}_0(\boldsymbol{y}(t))]_k=$$

$$-pw_i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}\tilde{\boldsymbol{A}}_0^i\boldsymbol{f}_0(\boldsymbol{y}(t)), \tag{21}$$

$$LV_5=2\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{q=1}^{m}\frac{1}{\beta_{jk}}\tilde{\boldsymbol{a}}_{qjk}^i\dot{\tilde{\boldsymbol{a}}}_{qjk}^i=$$

$$-\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{q=1}^{m}pw_i\tilde{\boldsymbol{a}}_{qjk}^i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}[\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t)))]_k=$$

$$-\sum_{q=1}^{m}\tilde{\boldsymbol{A}}_q^ipw_i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}\boldsymbol{f}_q(\boldsymbol{y}(t-h_q(t))), \tag{22}$$

$$LV_6=\delta(t)\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(t))-$$

$$(1-\dot{\delta}(t))\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s+2\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{1}{\chi_{jk}}\tilde{\boldsymbol{b}}_{jk}^i\dot{\tilde{\boldsymbol{b}}}_{jk}^i\leqslant$$

$$\delta\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(t))-(1-\sigma)\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s-$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}pw_i\tilde{\boldsymbol{b}}_{jk}^i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s=$$

$$\delta\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(t))-(1-\sigma)\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s-$$

$$pw_i(\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j)^{\mathrm{T}}|\boldsymbol{e}_j-(\boldsymbol{D}^i\boldsymbol{e}_{h_0})_j|^{p-2}\tilde{\boldsymbol{B}}^i\int_{t-\delta(t)}^{t}\boldsymbol{f}(\boldsymbol{y}(s))\mathrm{d}s. \tag{23}$$

结合式(18)~(23),可以得到

$$LV(t,i,\boldsymbol{e},\boldsymbol{e}_{h_0})\leqslant$$

$$pw_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\Big\{-\boldsymbol{C}^i\boldsymbol{e}(t)+$$

$$\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))+\sum_{q=1}^{m}\boldsymbol{A}_q^i\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q))+\boldsymbol{B}^i\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\Big\}+$$

$$\frac{1}{2}\operatorname{trace}[\boldsymbol{G}^{\mathrm{T}}(p(p-1)w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2})\boldsymbol{G}]+\sum_{j=1}^{N}\gamma_{ij}w_j|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p+$$

$$\int_{\mathbf{R}}(w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}+\boldsymbol{H}|^p-w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p)\nu\mathrm{d}z+$$

$$\delta\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(t))-(1-\sigma)\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{Q}_1\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s. \tag{24}$$

运用假设 2~4 以及引理 1、3 和 4 可得

$$(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))\leqslant$$

$$\frac{1}{2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_0^i(\boldsymbol{A}_0^i)^{\mathrm{T}}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})+\frac{1}{2}\tilde{\boldsymbol{f}}_0^{\mathrm{T}}(\boldsymbol{e}(t))\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))=$$

$$\frac{1}{2}[\boldsymbol{X}^{\mathrm{T}}\boldsymbol{A}_0^i(\boldsymbol{A}_0^i)^{\mathrm{T}}\boldsymbol{X}+L^2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)], \tag{25}$$

$$(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))\leqslant$$

$$\frac{1}{2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_0^i(\boldsymbol{A}_0^i)^{\mathrm{T}}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})+\frac{1}{2}\tilde{\boldsymbol{f}}_0^{\mathrm{T}}(\boldsymbol{e}(t))\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))=$$

$$\frac{1}{2}[\boldsymbol{X}^{\mathrm{T}}\boldsymbol{A}_0^i(\boldsymbol{A}_0^i)^{\mathrm{T}}\boldsymbol{X}+L^2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)], \tag{26}$$

$$\frac{1}{2}\mathrm{trace}[\boldsymbol{G}^{\mathrm{T}}(p(p-1)w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2})\boldsymbol{G}]\leqslant$$

$$\frac{1}{2}p(p-1)w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}[\boldsymbol{G}|\boldsymbol{e}(t)|^2+\boldsymbol{G}_1|\boldsymbol{e}_{h_1}(t)|^2+\cdots+$$

$$\boldsymbol{G}_q|\boldsymbol{e}_{h_q}(t)|^2+\boldsymbol{G}_\delta|\boldsymbol{e}_\delta(t)|^2], \tag{27}$$

$$\int_{\mathbf{R}}[w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}+\boldsymbol{H}|^p-w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p]\nu\mathrm{d}z\leqslant$$

$$w_i[\bar{\boldsymbol{\eta}}|\boldsymbol{e}(t)|^p+\boldsymbol{\eta}_0|\boldsymbol{e}_{h_0}(t)|^p+\boldsymbol{\eta}_1|\boldsymbol{e}_{h_1}(t)|^p+\cdots+$$

$$\boldsymbol{\eta}_q|\boldsymbol{e}_{h_q}(t)|^p+\boldsymbol{\eta}_\delta|\boldsymbol{e}_\delta(t)|^p], \tag{28}$$

并且有

$$\sum_{j=1}^{N}\gamma_{ij}w_j|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p=$$

$$\gamma_{ii}w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p+\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_j|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p=$$

$$-\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_i|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p+\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_j|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p\leqslant$$

$$\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_i(-(1-\mu)^{p-1}|\boldsymbol{e}|^p+\mu(1-\mu)^{p-1}|\boldsymbol{e}_{h_0}|^p)+$$

$$\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_j((1+\mu)^{p-1}(|\boldsymbol{e}|^p+\mu|\boldsymbol{e}_{h_0}|^p)). \tag{29}$$

使用 Young 不等式得

$$|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}|\boldsymbol{e}|^2\leqslant$$

$$\frac{p-2}{p}|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p+\frac{2}{p}|\boldsymbol{e}|^p\leqslant$$

$$\frac{p-2}{p}((1+\mu)^{p-1}(|\boldsymbol{e}|^p+\mu|\boldsymbol{e}_{h_0}|^p))+\frac{2}{p}|\boldsymbol{e}|^p=$$

$$\left(\frac{p-2}{p}(1+\mu)^{p-1}+\frac{2}{p}\right)|\boldsymbol{e}|^p+\left(\frac{p-2}{p}\mu(1+\mu)^{p-1}\right)|\boldsymbol{e}_{h_0}|^p, \tag{30}$$

$$\begin{aligned}&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}|\boldsymbol{e}_{h_0}|^2\leqslant\\&\frac{p-2}{p}|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^p+\frac{2}{p}|\boldsymbol{e}_{h_0}|^p\leqslant\\&\frac{p-2}{p}(1+\mu)^{p-1}(|\boldsymbol{e}|^p+\mu|\boldsymbol{e}_{h_0}|^p)+\frac{2}{p}|\boldsymbol{e}_{h_0}|^p=\\&\frac{p-2}{p}(1+\mu)^{p-1}|\boldsymbol{e}|^p+\left(\frac{p-2}{p}\mu(1+\mu)^{p-1}+\frac{2}{p}\right)|\boldsymbol{e}_{h_0}|^p,\end{aligned} \tag{31}$$

$$\begin{aligned}&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(-\boldsymbol{e}^{\mathrm{T}}\boldsymbol{C}^i\boldsymbol{e})\leqslant\\&(-|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2})\gamma|\boldsymbol{e}|^2\leqslant\\&\gamma|\boldsymbol{e}|^2(-(1-\mu)^{p-3}|\boldsymbol{e}|^{p-2}+\mu(1-\mu)^{p-3}|\boldsymbol{e}_{h_0}|^{p-2})=\\&\left(-\gamma(1-\mu)^{p-3}+\gamma\frac{2}{p}\mu(1-\mu)^{p-3}\right)|\boldsymbol{e}|^p+\gamma\frac{p-2}{p}\mu(1-\mu)^{p-3}|\boldsymbol{e}_{h_0}|^p,\end{aligned} \tag{32}$$

$$\begin{aligned}&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_0^i\tilde{\boldsymbol{f}}_0(\boldsymbol{e}(t))\leqslant\\&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}\left(\frac{1}{2}(\alpha(1+\mu)+L^2)|\boldsymbol{e}|^2+\frac{1}{2}\alpha\mu(1+\mu)|\boldsymbol{e}_{h_0}|^2\right),\end{aligned} \tag{33}$$

$$\begin{aligned}&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{A}_q^i\tilde{\boldsymbol{f}}_q(\boldsymbol{e}(t-h_q))\leqslant\\&|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}\left(\frac{1}{2}\beta(1+\mu)|\boldsymbol{e}|^2+\frac{1}{2}(\beta\mu(1+\mu)|\boldsymbol{e}_{h_0}|^2+L^2|\boldsymbol{e}_{h_q}|^2)\right).\end{aligned} \tag{34}$$

利用引理 1,计算可得

$$\begin{aligned}&(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{B}^i\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\leqslant\\&\frac{1}{2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{B}^i\boldsymbol{S}^{-1}(\boldsymbol{B}^i)^{\mathrm{T}}(\boldsymbol{e}-\boldsymbol{D}_i\boldsymbol{e}_{h_0})+\\&\left[\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\right]^{\mathrm{T}}\boldsymbol{S}\left[\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\right],\end{aligned} \tag{35}$$

其中, $\boldsymbol{S}$ 是正定对称矩阵.

再使用引理 5,我们有

$$\begin{aligned}&\left[\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\right]^{\mathrm{T}}\boldsymbol{S}\left[\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\right]\leqslant\\&\delta(t)\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\boldsymbol{S}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s\leqslant\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\delta\boldsymbol{S}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s,\end{aligned} \tag{36}$$

$$\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))\delta\boldsymbol{S}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\mathrm{d}s-\int_{t-\delta(t)}^{t}\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}(s))(1-\sigma)\boldsymbol{Q}\tilde{\boldsymbol{f}}(\boldsymbol{e}(s))\leqslant 0, \tag{37}$$

$$\begin{aligned}&\frac{1}{2}|\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0}|^{p-2}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})^{\mathrm{T}}\boldsymbol{B}^i\boldsymbol{S}^{-1}(\boldsymbol{B}^i)^{\mathrm{T}}(\boldsymbol{e}-\boldsymbol{D}^i\boldsymbol{e}_{h_0})\leqslant\\&\frac{1}{2}\vartheta(1+\mu)^{p-1}|\boldsymbol{e}|^p+\frac{1}{2}\vartheta\mu(1+\mu)^{p-1}|\boldsymbol{e}_{h_0}|^p.\end{aligned} \tag{38}$$

把式(25)~(38)代入式(10),通过计算可得

$$
\begin{aligned}
LV \leqslant & [w_i(H_1+H_2+H_3+H_4)+H_5+\bar{\eta}]|\boldsymbol{e}|^p + \\
& [w_i(J_1+J_2+J_3+J_4)+J_5+\boldsymbol{\eta}_0]|\boldsymbol{e}_{h_0}|^p + \\
& w_i(L^2+\boldsymbol{\eta}_1+G_1)|\boldsymbol{e}_{h_1}|^p+\cdots+w_i(L^2+\boldsymbol{\eta}_m+G_m)|\boldsymbol{e}_{h_m}|^p,
\end{aligned} \tag{39}
$$

其中

$$
\begin{aligned}
H_1 &= -\gamma p(1-\mu)^{p-3}+2\gamma\mu(1-\mu)^{p-3}+\frac{1}{2}p\vartheta(1+\mu)^{p-1}, \\
H_2 &= \frac{1}{2}((\alpha+\beta)(1+\mu)+L^2+(p-1)\bar{G})((p-2)(1+\mu)^{p-1}+2), \\
H_3 &= \frac{1}{2}(\mu(1+\mu)(\alpha+\beta)+L^2+(p-1)G_0)(p-2)(1+\mu)^{p-1}, \\
H_4 &= \frac{1}{2}(p-1)[m(p-2)(1+\mu)^{p-1}], \\
H_5 &= \sum_{j=1,j\neq i}^{N}\gamma_{ij}w_i(-(1-\mu)^{p-1})+\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_j(1+\mu)^{p-1} \leqslant \\
& -a_1w_i(1-\mu)^{p-1}+a_2w_j(1+\mu)^{p-1}+(1+\mu)^{p-1}\sum_{j=1}^{N}\gamma_{ij}q_j, \\
J_1 &= \gamma(p-2)\mu(1-\mu)^{p-3}+\frac{1}{2}p\vartheta\mu(1+\mu)^{p-1}, \\
J_2 &= \frac{1}{2}((\alpha+\beta)(1+\mu)+L^2+(p-1)\bar{G})(p-2)\mu(1+\mu)^{p-1}, \\
J_3 &= \frac{1}{2}(\mu(1+\mu)(\alpha+\beta)+L^2+(p-1)G_0)((p-2)\mu(1+\mu)^{p-1}+2), \\
J_4 &= \frac{1}{2}(p-1)[m\mu(p-2)(1+\mu)^{p-1}], \\
J_5 &= \sum_{j=1,j\neq i}^{N}\gamma_{ij}w_i\mu(1-\mu)^{1-p}+\mu\sum_{j=1,j\neq i}^{N}\gamma_{ij}w_j \leqslant \\
& a_2c_2\mu(1-\mu)^{p-1}+b_1\mu, \\
a_1 &= \min_{i\in S}\sum_{k=1,k\neq i}^{N}\gamma_{ik},\ a_2=\max_{i\in S}\sum_{k=1,k\neq i}^{N}\gamma_{ik},\ b_1=\min_{i\in S}\sum_{k=1,k\neq i}^{N}\gamma_{ik}q_k,\ b_2=\max_{i\in S}\sum_{k=1,k\neq i}^{N}\gamma_{ik}q_k.
\end{aligned}
$$

整理可得

$$
\begin{aligned}
LV \leqslant & \Big((H_1+H_2+H_3+H_4-a_1(1-\mu)^{p-1}+a_2(1+\mu)^{p-1})w_i + \\
& (1+\mu)^{p-1}\sum_{j=1}^{N}\gamma_{ij}q_j+\bar{\eta}\Big)|\boldsymbol{e}|^p + \\
& ((J_1+J_2+J_3+J_4)w_i+a_2c_2\mu(1-\mu)^{p-1}+b_1\mu+\boldsymbol{\eta}_0)|\boldsymbol{e}_{h_0}|^p + \\
& w_i(L^2+\boldsymbol{\eta}_1+G_1)|\boldsymbol{e}_{h_1}|^p+\cdots+w_i(L^2+\boldsymbol{\eta}_m+G_m)|\boldsymbol{e}_{h_m}|^p = \\
& \Big(w_i\boldsymbol{\kappa}+(1+\mu)^{p-1}\sum_{j=1}^{N}\gamma_{ij}q_j+\bar{\eta}\Big)|\boldsymbol{e}|^p+\bar{w}\sum_{q=1}^{m}(L^2+\boldsymbol{\eta}_q+G_q)|\boldsymbol{e}_{h_q}|^p = \\
& \bar{\iota}|\boldsymbol{e}|^p+\sum_{q=0}^{m}\boldsymbol{\iota}_p|\boldsymbol{e}_{h_p}|^p,
\end{aligned} \tag{40}
$$

其中

$$\kappa = H_1 + H_2 + H_3 + H_4 - a_1(1-\mu)^{p-1} + a_2(1+\mu)^{p-1},$$

$$\bar{\iota} = w_i\kappa + (1+\mu)^{p-1}\sum_{j=1}^{N}\gamma_{ij}q_j + \bar{\eta},$$

$$\iota_0 = (J_1 + J_2 + J_3 + J_4)w_i + a_2c_2\mu(1-\mu)^{p-1} + b_1\mu + \eta_0,\ \iota_q = L^2 + \eta_q + G_q.$$

根据广义 Dynkin 定理可得

$$E|\boldsymbol{e}(t)|^p \leqslant EV(0,\boldsymbol{\xi}(0)) + E\int_0^t LV(s,\boldsymbol{e}(s),\boldsymbol{e}_{h_1}(s),\cdots,\boldsymbol{e}_{h_m}(s))\mathrm{d}s \leqslant$$

$$EV(0,\boldsymbol{\xi}(0)) + E\int_0^t\left[\bar{\iota}|\boldsymbol{e}|^p + \sum_{q=0}^{m}\iota_q|\boldsymbol{e}_{h_q}|^p\right]\mathrm{d}s, \tag{41}$$

$$\int_0^t|\boldsymbol{e}_{h_q}(s)|^p\mathrm{d}s = \int_{-h_q}^{t-h_q}\frac{1}{1-\dot{h}_q(s)}|\boldsymbol{e}(s)|^p\mathrm{d}s \leqslant \frac{1}{1-\hat{h}}\int_{-\bar{h}}^{t}|\boldsymbol{e}(s)|^p\mathrm{d}s =$$

$$\frac{1}{1-\hat{h}}\int_{-\bar{h}}^{0}|\boldsymbol{e}(s)|^p\mathrm{d}s + \frac{1}{1-\hat{h}}\int_0^t|\boldsymbol{e}(s)|^p\mathrm{d}s \leqslant$$

$$\frac{\bar{h}}{1-\hat{h}}\max_{-\bar{h}\leqslant s\leqslant 0}|\boldsymbol{\xi}(s)|^p + \frac{1}{1-\hat{h}}\int_0^t|\boldsymbol{e}(s)|^p\mathrm{d}s, \tag{42}$$

$$E|\boldsymbol{e}(t)|^p \leqslant EV(0,\boldsymbol{\xi}(0)) + \frac{\bar{h}}{1-\hat{h}}\sum_{q=0}^{m}\iota_q\max_{-\bar{h}\leqslant s\leqslant 0}E|\boldsymbol{\xi}(s)|^p +$$

$$\int_0^t\left(\bar{\iota} + \frac{1}{1-\hat{h}}\sum_{q=0}^{m}\iota_q\right)E|\boldsymbol{e}(s)|^p\mathrm{d}s =$$

$$\upsilon_1 + \int_0^t\upsilon_2E|\boldsymbol{e}(s)|^p\mathrm{d}s, \tag{43}$$

其中

$$\upsilon_1 = EV(0,\boldsymbol{\xi}(0)) + \frac{\bar{h}}{1-\hat{h}}\sum_{q=0}^{m}\iota_q\max_{-\bar{h}\leqslant s\leqslant 0}E|\boldsymbol{\xi}(s)|^p,$$

$$\upsilon_2 = \bar{\iota} + \frac{1}{1-\hat{h}}\sum_{q=0}^{m}\iota_q,\ \upsilon_1 > 0,\ \upsilon_2 < 0.$$

由 $E|\boldsymbol{e}(t)|^p \leqslant \upsilon_1 e^{\upsilon_2 t}$ 可得，$\limsup_{t\to\infty}\frac{1}{t}\lg(E|\boldsymbol{e}(t,\xi)|^p) \leqslant \upsilon_2 < 0$.

因此误差系统(3)是 p 阶指数稳定的，可得响应系统(2)和驱动系统(1)是 p 阶自适应指数同步的.

3 数值仿真

我们考虑系统(1)~(3). 令 $S=\{1,2\}$ 为 Markov 链 $\{r(t):t\geqslant 0\}$ 的状态空间，$\boldsymbol{\Gamma} = \begin{bmatrix}-1.1 & 1.1\\ 0.6 & -0.6\end{bmatrix}$，其 Markov 链如图 1 所示.

网络参数给定如下：

$$\boldsymbol{D}^1 = \begin{bmatrix}0.3 & 0\\ 0 & 0.2\end{bmatrix},\ \boldsymbol{D}^2 = \begin{bmatrix}0.1 & 0\\ 0 & 0.15\end{bmatrix},\ \boldsymbol{C}^1 = \hat{\boldsymbol{C}}^1 = \begin{bmatrix}0.9 & 0\\ 0 & 1\end{bmatrix},\ \boldsymbol{C}^2 = \hat{\boldsymbol{C}}^2 = \begin{bmatrix}1 & 0\\ 0 & 0.9\end{bmatrix},$$

$$A_0^1 = \hat{A}_0^1 = \begin{bmatrix} 0.03 & -0.04 \\ -0.06 & 0.05 \end{bmatrix},\ A_0^2 = \hat{A}_0^2 = \begin{bmatrix} -0.03 & 0.02 \\ 0.03 & -0.06 \end{bmatrix},$$

$$A_1^1 = \hat{A}_1^1 = \begin{bmatrix} 0.06 & -0.03 \\ 0.04 & -0.06 \end{bmatrix},\ A_1^2 = \hat{A}_1^2 = \begin{bmatrix} 0.04 & -0.07 \\ 0.03 & 0.02 \end{bmatrix},$$

$$A_2^1 = \hat{A}_2^1 = \begin{bmatrix} -0.07 & 0.03 \\ 0.06 & -0.04 \end{bmatrix},\ A_2^2 = \hat{A}_2^2 = \begin{bmatrix} -0.04 & -0.03 \\ 0.05 & -0.03 \end{bmatrix},$$

$$B^1 = \hat{B}^1 = \begin{bmatrix} 0.05 & 0.02 \\ -0.03 & 0.05 \end{bmatrix},\ B^2 = \hat{B}^2 = \begin{bmatrix} 0.04 & 0.01 \\ -0.05 & 0.02 \end{bmatrix}.$$

神经元激活函数选择如下$f(x(t)) = \tanh(x(t))$，取$h_0 = 0.1, h_1 = 0.4, h_2 = 0.7$，令扰动为 Lévy 噪声.Lévy 噪声如图 2 和 3 所示.

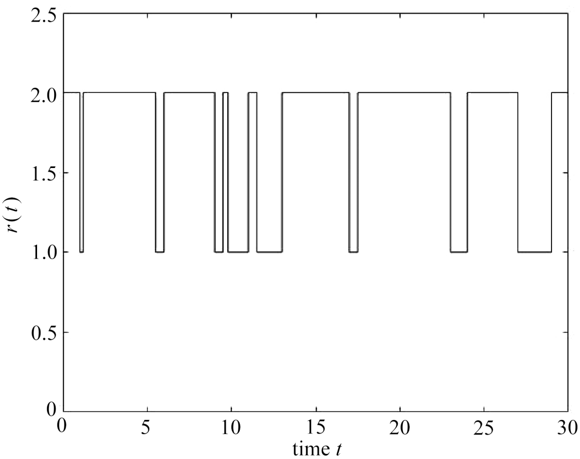

图 1 Markov 链

Fig. 1 The Markov chain

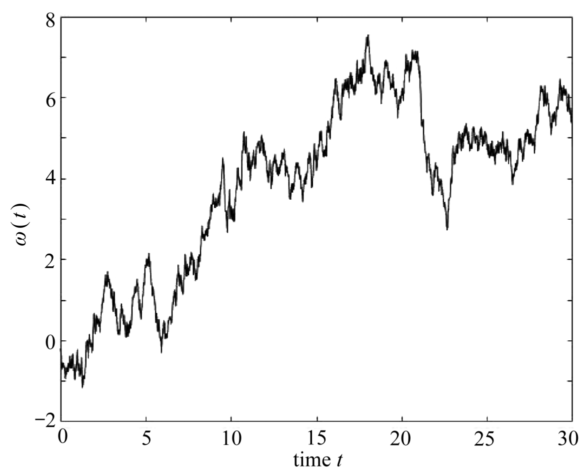

图 2 Brown 运动

Fig. 2 The Brownian motion

连续的噪声强度函数如下：

$$G(t,1,e,e_{h_1},e_{h_2}) = [0.3e + 0.2e_{h_2},\ 0.5e_{h_1}],$$

$$\boldsymbol{G}(t,2,\boldsymbol{e},\boldsymbol{e}_{h_1},\boldsymbol{e}_{h_2}) = [0.4\boldsymbol{e}_{h_2},\ 0.1\boldsymbol{e} + 0.2\boldsymbol{e}_{h_1}];$$

不连续的噪声强度函数如下:

$$\boldsymbol{H}(t,1,z,\boldsymbol{e},\boldsymbol{e}_{h_1},\boldsymbol{e}_{h_2}) = z(\boldsymbol{e} + \boldsymbol{e}_{h_1} + \boldsymbol{e}_{h_2})/10,$$

$$\boldsymbol{H}(t,2,z,\boldsymbol{e},\boldsymbol{e}_{h_1},\boldsymbol{e}_{h_2}) = z(\boldsymbol{e} + \boldsymbol{e}_{h_1} + \boldsymbol{e}_{h_2})/10.$$

通过上面参数易得

$$\alpha = 0.186\,3,\ \beta_1 = 0.204\,4,\ \beta_2 = 0.228\,5,\ \mu = 0.2,\ L = 1,$$

$$\bar{\iota} = -0.110\,4,\ \iota_0 = 0.187\,2,\ \iota_1 = 0.003,\ \iota_2 = 0.005.$$

假设 1~4 成立,且符合条件(14).因此,依据定理 2,响应系统(2)和驱动系统(1)是 p 阶自适应指数同步.误差系统(3)的状态轨迹见图 4,根据图 4 可以看出响应系统(2)和驱动系统(1)是同步的.其中控制率的状态变化见图 5.本文的结果将文献[9]中的扰动由连续的 Gauss 噪声扩展到同时包含连续和不连续部分的 Lévy 噪声,此外,还将文献[16]中的离散时滞推广到了离散和分布都存在的情况.

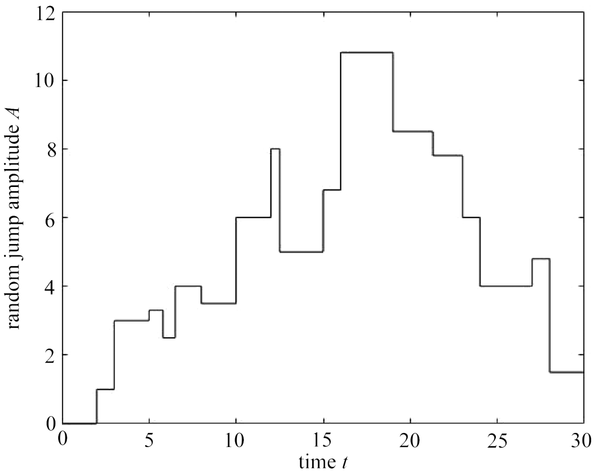

图 3 Poisson 点过程

Fig. 3 The Poisson point process

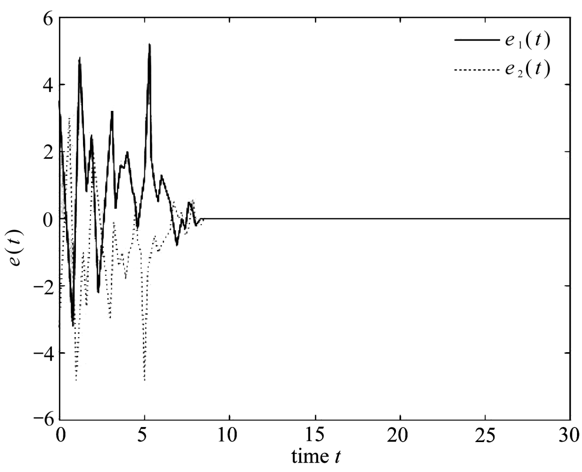

图 4 误差系统的状态曲线

Fig. 4 The state trajectory of the error system

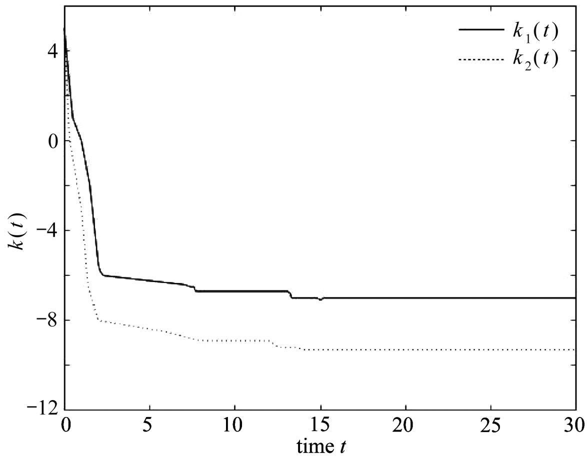

图 5　控制增益的更新率

Fig. 5　The update rate of the gain controller

4　结　　论

本文针对基于 Lévy 噪声的混合时滞中立型神经网络系统，通过线性矩阵不等式方法和 Lyapunov 稳定性理论，给出了系统的自适应同步准则和控制器的更新率。所提出的 Lévy 噪声，使得网络里的噪声干扰不仅包含了连续的扰动，而且含有不连续的突触噪声。同时，中立型神经网络系统中的时滞是包含离散和分布两部分的混合时滞，考虑了更多的时滞信息，所得的结论相对具有较小的保守性。最后，本文提供了一个数值实例来验证所提出方法的有效性。

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Adaptive Synchronization of Neutral Neural Networks With Mixed Delays and Lévy Noises

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Abstract: The problem of feedback controllers designed to achieve adaptive synchronization was investigated for neutral neural networks with mixed delays and Lévy noises. The noise disturbance in the neural network model was driven by the Lévy stochastic process consisting of the Gaussian process and the Poisson point process, and involving continuous disturbances as well as discontinuous synaptic noises. Based on the Lyapunov functional, the Itôs formula and the inequality analysis technique, the criteria to ensure adaptive stabilization for the error system were built. Moreover, the update rate of the feedback controller was given to enhance the adaptive synchronization of the response system and the drive system. Results of a simulation example show the effectiveness of the theoretical analysis.

Key words: neutral neural network; Lévy noise; adaptive synchronization; mixed delays

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