一类催化反应 Robin 问题的渐近解*

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摘要: 研究了一类非线性催化反应微分方程 Robin 问题.在一定的条件下,先利用摄动方法求出了原 Robin 问题的外部解,然后用伸长变量和幂级数理论分别构造了解的第一和第二边界层校正项,从而得到了 Robin 问题解的形式渐近展开式.最后利用微分不等式理论,证明了问题解的渐近表示式的一致有效性.

关 键 词: 催化反应; Robin 问题; 奇异摄动

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引 言

非线性问题在学术界的研究中十分活跃[1].近来出现了许多渐近方法.许多学者,例如,De Jager 等[2]、Barbu 等[3]、Chang 等[4]、Martinez 等[5]、Kellogg 等[6]、Tian 等[7]、Skrynnikov[8]、Samusenko[9]做了许多工作.Mo 和 Chen 等用伸长变量、多重尺度、匹配和微分不等式等方法研究了一类奇摄动非线性微分边值问题、反应扩散问题、椭圆型问题、双曲型初始-边值问题、生物数学问题、冲击波问题、孤立子问题和大气物理问题等[10-24].本文将讨论一类催化反应过程中的一类非线性问题,利用微分不等式等理论,得到了问题解的渐近估计.

现研究以下在催化反应理论中出现的非线性微分方程 Robin 边值问题[5-6]:

$$\varepsilon^2\frac{\mathrm{d}^2z}{\mathrm{d}x^2}-\mu\frac{\mathrm{d}z}{\mathrm{d}x}=\prod_{k=1}^{m}(z-y_k(x)),\qquad a<x<b, \tag{1}$$

$$z(a,\varepsilon)-\varepsilon\frac{\mathrm{d}z}{\mathrm{d}x}(a,\varepsilon)=A, \tag{2}$$

$$z(b,\varepsilon)+\mu\frac{\mathrm{d}z}{\mathrm{d}x}(b,\varepsilon)=B, \tag{3}$$

其中式(1)为压片内催化的扩散与反应间质量平衡的方程, x 是压片上的位置,z 是反应的浓度,$y_k(x)\in C^2[a,b]\,(k=1,2,\cdots,m)$ 为与催化浓度相关位置的函数,ε,μ 为摄动参数,它们与

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扩散系数和反应速率相关.本文将应用奇异摄动方法构造催化反应问题(1)~(3)的形式渐近解,然后利用微分不等式理论证明该渐近解的一致有效性.

假设:

(H1) 正的小参数 ε,μ 满足 $0 < \sigma = \dfrac{\varepsilon}{\mu} \ll 0$;

(H2) 存在正常数 c, 使得 $\dfrac{\partial}{\partial z}\left[\prod_{k=1}^{m}(z - y_k(x))\right] \leqslant -c$.

1 外部解

由奇异摄动理论,催化反应奇摄动问题(1)~(3)的退化情形为

$$\prod_{k=1}^{m}(z - y_k(x)) = 0, \qquad a < x < b. \tag{4}$$

显然退化方程(4)的解为

$$z(x) = y_k(x), \qquad a < x < b,\ 1 \leqslant k \leqslant m.$$

不失一般性,下面不妨考虑退化解为

$$\bar{z}(x) = y_1(x), \qquad a < x < b \tag{5}$$

的情形.

设 Robin 问题(1)~(3)的外部解 $Z(x)$ 的形式展开式为

$$Z(x) \sim \sum_{i,j=0}^{\infty} Z_{ij}(x)\varepsilon^i\mu^j. \tag{6}$$

将式(6)代入式(1),等式的右端按 ε,μ 的幂展开,比较 $\varepsilon^0\mu^0$ 的同次幂项.考虑到式(5),显然可得

$$Z_{00}(x) = y_1(x), \qquad a \leqslant x \leqslant b. \tag{7}$$

将式(6)、(7)代入式(1),比较 $\varepsilon^i\mu^j(i + j \neq 0)$ 的同次幂项得

$$\frac{\mathrm{d}^2 Z_{(i-2)j}}{\mathrm{d}x^2} - \frac{\mathrm{d}Z_{i(j-1)}}{\mathrm{d}x} = F_{ij}, \qquad i,\ j = 0,1,\cdots,\ i + j \neq 0, \tag{8}$$

其中

$$F_{ij} = \frac{1}{i!\ j!}\left[\frac{\partial^{i+j}}{\partial\varepsilon^i\partial\mu^j}\left(\left(\sum_{r,s=0}^{\infty} Z_{ij}(x)\varepsilon^r\mu^s - y_1(x)\right)\prod_{k=2}^{m}\left(\sum_{r,s=0}^{\infty} Z_{ij}(x)\varepsilon^r\mu^s - y_k(x)\right)\right)\right]_{\varepsilon,\mu=0}$$

为依次已知的函数.

假设带有负下标的项均设为零.由方程(8),可依次得到 $Z_{ij}(x)(i,\ j = 0,1,\cdots,i + j \neq 0)$.再将求得的 $Z_{ij}(x)(i,\ j = 0,1,\cdots)$ 代入式(6),便得到了催化反应微分方程 Robin 边值问题(1)~(3)的外部解.但由式(6)决定的外部解未必满足 Robin 边界条件式(2)和(3).为此我们还需要分别构造在 $x = a$ 和 $x = b$ 邻域的边界层校正项.

2 第一边界层校正项

由奇异摄动理论可知[4],奇异摄动微分方程 Robin 边值问题(1)~(3)解的第一边界层校正项 U 由式(1)、(2)来决定.先作伸长变量变换:

$$\tau = \frac{x - a}{\mu}, \tag{9}$$

并设

$$z = Z + U. \tag{10}$$

将式(9)、(10)代入式(1)、(2)：

$$\sigma^2 \frac{d^2U}{d\tau^2} - \frac{dU}{d\tau} = \prod_{k=1}^{m} [Z + U - y_k(a + \mu\tau)] - \sigma^2 \frac{d^2Z}{d\tau^2} + \frac{dZ}{d\tau}, \tag{11}$$

$$U(0) - \frac{dU}{d\tau}(0) = A - \left(Z - \frac{dZ}{d\tau}\right)_{x=a}. \tag{12}$$

再设

$$U(\tau) \sim \sum_{i,j=0}^{\infty} U_{ij}(\tau)\varepsilon^i\sigma^j. \tag{13}$$

将式(13)代入式(11)、(12)，等式右端按 ε,σ 的幂展开，比较 $\varepsilon^i\mu^j$ 的同次幂项，可得

$$\frac{dU_{ij}}{d\tau} = \frac{d^2U_{i(j-2)}}{d\tau^2} - \bar{F}_{ij}, \qquad i, j = 0,1,\cdots, \tag{14}$$

$$U_{ij}(0) - \frac{dU_{ij}}{d\tau}(0) = \left(A - Z_{ij} + \frac{dZ_{ij}}{d\tau}\right)_{x=a}, \qquad i, j = 0,1,\cdots, \tag{15}$$

其中

$$\bar{F}_{ij} = \frac{1}{i!\ j!}\left[\frac{\partial^{i+j}}{\partial\varepsilon^i\partial\sigma^j}\prod_{k=1}^{m}\left[Z + \sum_{r,s=0}^{\infty} U_{rs}(\tau)\varepsilon^r\sigma^s - y_k(a + \mu\tau)\right] - \sigma^2\frac{d^2Z}{d\tau^2} + \frac{dZ}{d\tau}\right]_{\varepsilon=\sigma=0},$$

而

$$Z = \sum_{i,j=0}^{\infty} Z_{ij}(a + \mu\tau)\varepsilon^i(\varepsilon/\sigma)^j.$$

由式(14)、(15)，便可依次地求出解 $U_{ij}(\tau)(i, j = 0,1,\cdots)$.

由假设不难知道 $U_{ij}(\tau)(i, j = 0,1,\cdots)$ 具有如下性态：

$$U_{ij}(\tau) = O(\exp(-\delta_{ij}\tau)) = O\left(\exp\left(-\delta_{ij}\frac{x - a}{\mu}\right)\right), \qquad i, j = 0,1,\cdots, \tag{16}$$

其中 $\delta_{ij}(i, j = 0,1,\cdots)$ 为充分小的正常数.再将 $U_{ij}(\tau)(i, j = 0,1,\cdots)$ 代入式(13)，便得到奇异摄动微分方程 Robin 边值问题(1)~(3)解的第一边界层校正项 U.

3 第二边界层校正项

由奇异摄动理论，奇异摄动微分方程 Robin 边值问题(1)~(3)解的第二边界层校正项 V 由式(1)、(3)来决定.作伸长变量变换：

$$\rho = \frac{b - x}{\sigma^2\mu}, \tag{17}$$

并设

$$z = Z + V. \tag{18}$$

将式(17)、(18)代入式(1)、(3)：

$$\frac{d^2V}{d\rho^2} + \frac{dV}{d\rho} = \sigma^2\prod_{k=1}^{m}[Z + V - y_k(b - \sigma^2\mu\rho)] - \frac{d^2Z}{d\rho^2} + \frac{dZ}{d\rho}, \tag{19}$$

$$V(0) - \frac{dV}{d\rho}(0) = \left(B - Z + \frac{dZ}{d\rho}\right)_{x=b}. \tag{20}$$

再设

$$V(\rho)\sim\sum_{i,j=0}^{\infty}V_{ij}(\rho)\sigma^i\mu^j. \tag{21}$$

将式(21)代入式(19)、(20),等式右端按 σ,μ 的幂展开,比较 $\sigma^i\mu^j$ 的同次幂项,可得

$$\frac{d^2V_{ij}}{d\rho^2}+\frac{dV_{ij}}{d\rho}=\tilde{F}_{ij},\qquad i,j=0,1,\cdots, \tag{22}$$

$$V_{ij}(0)-\frac{dV_{ij}}{d\rho}(0)=\left(B-Z_{ij}+\frac{dZ_{ij}}{d\rho}\right)_{x=b},\qquad i,j=0,1,\cdots, \tag{23}$$

其中

$$\tilde{F}_{ij}=\frac{1}{i!\,j!}\left[\frac{\partial^{i+j}}{\partial\sigma^i\partial\mu^j}\sigma^2\prod_{k=1}^{m}\left[Z+\sum_{r,s=0}^{\infty}V_{rs}(\tau)\sigma^r\mu^s-y_k(b-\sigma^2\mu\rho)\right]-\frac{d^2Z}{d\rho^2}+\frac{dZ}{d\rho}\right]_{\sigma=\mu=0},$$

而

$$Z=\sum_{i,j=0}^{\infty}Z_{ij}(b-\sigma^2\mu\rho)\sigma^i\mu^j.$$

由式(22)、(23),便可依次地求出具有如下性态

$$V_{ij}(\rho)=O(\exp(-\bar{\delta}_{ij}\rho))=O\left(\exp\left(-\bar{\delta}_{ij}\frac{b-x}{\sigma^2\mu}\right)\right),\qquad i,j=0,1,\cdots \tag{24}$$

的解 $V_{ij}(\tau)(i,j=0,1,\cdots)$,其中 $\bar{\delta}_{ij}(i,j=0,1,\cdots)$ 为充分小的正常数.再将 $V_{ij}(\rho)(i,j=0,1,\cdots)$ 代入式(21),便得到奇异摄动微分方程 Robin 边值问题(1)~(3)解的第二边界层校正项 V.

由式(6)、(13)、(21)和式(10)、(18),可以得到奇异摄动微分方程 Robin 边值问题(1)~(3)解 $z(x)$ 的如下形式渐近展开式:

$$z(x)\sim\sum_{i,j=0}^{\infty}Z_{ij}(x)\varepsilon^i\mu^j+\sum_{i,j=0}^{\infty}U_{ij}(\tau)\varepsilon^i\sigma^j+\sum_{i,j=0}^{\infty}V_{ij}(\rho)\sigma^i\mu^j,$$
$$a\leqslant x\leqslant b,\ 0<\varepsilon,\mu,\sigma=\varepsilon/\mu\ll 1. \tag{25}$$

4 渐近解及其一致有效性

现讨论由式(25)表示的微分方程 Robin 边值问题(1)~(3)解的一致有效性.

定理 1 在假设(H1)、(H2)下,奇异摄动非线性催化反应微分方程 Robin 问题(1)~(3)有解 $z(x)\in C^2[a,b]$,并具有如下形式的一致有效的渐近展开式:

$$z(x)=\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j+\sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j+\sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j+O(\lambda^{2M+1}),$$
$$a\leqslant x\leqslant b,\ \lambda=\max(\varepsilon,\mu,\sigma),\ 0<\varepsilon,\mu,\sigma=\varepsilon/\mu\ll 1. \tag{26}$$

证明 首先在 $x\in[a,b]$ 上作辅助函数:

$$\alpha(x)=\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j+\sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j+\sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j-\gamma\lambda^{2M+1}, \tag{27}$$

$$\beta(x)=\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j+\sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j+\sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j+\gamma\lambda^{2M+1}, \tag{28}$$

其中 γ 为足够大的待定正常数.

显然

$$\alpha(x) \leqslant \beta(x), \qquad x \in [a,b]. \tag{29}$$

由式(27),并考虑到式(12)和(15),并由假设,对足够小的 ε,μ, 存在正常数 D_1 使得

$$\alpha(a) - \varepsilon\frac{\mathrm{d}\alpha}{\mathrm{d}x}(a) - A \leqslant U(0) - \frac{\mathrm{d}U}{\mathrm{d}\tau}(0) - A + \left(Z - \frac{\mathrm{d}Z}{\mathrm{d}\tau}\right)_{x=a} + \sum_{i,j=0,i+j\neq 0}^{M}\left[U_{ij}(0) - \frac{\mathrm{d}U_{ij}}{\mathrm{d}\tau}(0) - \left(A - Z_{ij} + \frac{\mathrm{d}Z_{ij}}{\mathrm{d}\tau}\right)_{x=a}\right]\varepsilon^i\mu^i + D_1\lambda^{2M+1} - \gamma\lambda^{2M+1} = (D_1 - \gamma)\lambda^{2M+1}.$$

选取 $\gamma \geqslant D_1$, 由上式有

$$\alpha(a) - \varepsilon\frac{\mathrm{d}\alpha}{\mathrm{d}x}(a) \leqslant A, \tag{30}$$

同理可得

$$\beta(a) - \varepsilon\frac{\mathrm{d}\beta}{\mathrm{d}x}(a) \geqslant A. \tag{31}$$

由式(27),并考虑到式(20)、(23),并由假设,对足够小的 ε,μ, 存在正常数 D_2, 使得

$$\alpha(b) + \mu\frac{\mathrm{d}\alpha}{\mathrm{d}x}(b) - B \leqslant V(0) - \frac{\mathrm{d}V}{\mathrm{d}\rho}(0) - \left(B - Z + \frac{\mathrm{d}Z}{\mathrm{d}\rho}\right)_{x=b} - \gamma\lambda^{2M+1} + \sum_{i,j=0,i+j\neq 0}^{M}\left[V_{ij}(0) - \frac{\mathrm{d}V_{ij}}{\mathrm{d}\tau}(0) - \left(B - Z_{ij} + \frac{\mathrm{d}Z_{ij}}{\mathrm{d}\rho}\right)_{x=b}\right]\varepsilon^i\mu^i + D_2\lambda^{2M+1} = (D_2 - \gamma)\lambda^{2M+1}.$$

选取 $\gamma \geqslant D_2$, 由上式有

$$\alpha(b) + \mu\frac{\mathrm{d}\alpha}{\mathrm{d}x}(b) \leqslant B, \tag{32}$$

同理可得

$$\beta(b) + \mu\frac{\mathrm{d}\beta}{\mathrm{d}x}(b) \geqslant B. \tag{33}$$

下面来证明

$$\varepsilon^2\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2} - \mu\frac{\mathrm{d}\alpha}{\mathrm{d}x} - \prod_{k=1}^{m}(\alpha - y_k(x)) \geqslant 0, \qquad a < x < b, \tag{34}$$

$$\varepsilon^2\frac{\mathrm{d}^2\beta}{\mathrm{d}x^2} - \mu\frac{\mathrm{d}\beta}{\mathrm{d}x} - \prod_{k=1}^{m}(\beta - y_k(x)) \geqslant 0, \qquad a < x < b. \tag{35}$$

事实上,由关系式(27)及式(16)、(24),对于足够小的 ε,μ, 存在一个正常数 D_3, 使下式成立:

$$\varepsilon^2\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2} - \mu\frac{\mathrm{d}\alpha}{\mathrm{d}x} - \prod_{k=1}^{m}(\alpha - y_k(x)) \geqslant$$
$$\varepsilon^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j + \sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j + \sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j\right] - \mu\left[\frac{\mathrm{d}}{\mathrm{d}x}\left[\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j + \sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j + \sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j\right]\right] - \prod_{k=1}^{m}\left(\left(\sum_{i,j=0}^{M}Z_{ij}(x)\varepsilon^i\mu^j + \sum_{i,j=0}^{M}U_{ij}(\tau)\varepsilon^i\sigma^j + \sum_{i,j=0}^{M}V_{ij}(\rho)\sigma^i\mu^j - \gamma\lambda^{2M+1}\right) - y_k(x)\right) \geqslant$$

$$[z - y_1(x)]\prod_{k=2}^{m}(z - y_k(x)) + \sum_{i,j,i+j\neq 0}^{M}\left(\frac{d^2 Z_{(i-2)j}}{dx^2} - \frac{dZ_{i(j-1)}}{dx} - F_{ij}\right)\varepsilon^i\mu^j +$$
$$\sum_{i,j=0}^{M}\left(\frac{dU_{ij}}{d\tau} - \frac{d^2 U_{i(j-2)}}{d\tau^2} + \bar{F}_{ij}\right)\varepsilon^i\sigma^i + \sum_{i,j=0}^{M}\left(\frac{d^2 V_{ij}}{d\rho^2} + \frac{dV_{ij}}{d\rho} - \tilde{F}_{ij}\right)\sigma^i\mu^j +$$
$$(D_3 + c)\lambda^{2M+1} - \gamma\lambda^{2M+1} = (D_3 + c - \gamma)\lambda^{2M+1}.$$

选取 $\gamma \geqslant D_3 + c$，由上式知式(34)成立.

同理可证式(35)成立.

由关系式(29)~(35)和微分不等式理论[4]，奇异摄动非线性催化反应微分方程 Robin 问题(1)~(3)存在一个解 $z(x) \in C^2[a,b]$，并由式(27)、(28)，有

$$\alpha(x) \leqslant z(x) \leqslant \beta(x), \qquad a \leqslant x \leqslant b.$$

于是问题解 $z(x)$ 的一致有效的渐近估计式(26)成立.定理证毕.

5　结　　论

近来对非线性问题优化了许多渐近方法.许多学者做了很多的工作.本文讨论了一类催化反应过程中的一类非线性 Robin 边值问题，利用微分不等式等理论，得到了问题解的渐近解析表示式并证明了它的一致有效性.本文所述的内容思路明确、方法简单.并且还能通过该渐近解的表示式，继续进行解析运算，从而进一步得到相关物理量的渐近表示式.这是运用简单的数值模拟计算所不能的.因此本文通过对一类奇异摄动非线性催化反应微分方程 Robin 问题求得其渐近解的研讨，具有较广泛的研究前景.

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Asymptotic Solutions to a Class of Catalytic Reaction Robin Problems

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Abstract: A class of Robin problems of nonlinear catalytic reaction differential equations were studied. Firstly, under the suitable conditions, the outer solution to the original Robin problem was obtained with the perturbation method. Then by means of the stretched variable and the power series, the 1st and 2nd boundary layer corrective terms were constructed respectively, and the formal asymptotic expansion was structured. Finally, based on the theory of differential inequalities the formal asymptotic expression of the solution to the Robin problem was given. Finally, the uniform validity of the asymptotic expression of the solution to problem was proved.

Key words: catalytic reaction; Robin problem; singular perturbation

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