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流通量间断守恒高阶交通流模型及其数值模拟*

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摘要: 在非均匀道路条件下,推广了各向异性守恒高阶交通流模型(CHO 模型),获得流通量间断 CHO 模型,并基于其 Riemann 不变量性质,运用局部简化方法及 δ 映射算法,设计了求解流通量间断 CHO 模型的一阶 Godunov、EO(Engquist-Osher)和 LF(Lax-Friedrichs)等数值格式.通过数值模拟表明流通量间断 CHO 模型是合理有效的,它可以描述平衡态和非平衡态交通流,相对于流通量间断 LWR(Lighthill-Whitham-Richards)模型,其能更好地刻画实际交通现象.

关 键 词: 流通量间断 CHO 模型; Riemann 不变量; 局部简化; δ 映射

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引 言

流通量间断双曲守恒律方程的一般形式为

$$u_t + f(u,\theta(x))_x = s(u,\theta(x)), \tag{1}$$

解变量 $u = u(x,t)$ 为未知的标量或矢量,流通量函数 $f(u,\theta(x))$ 依赖于与空间坐标 x 有关的标量或矢量 $\theta(x)$, $s(u,\theta(x))$ 为源项.在很多实际问题中,$\theta(x)$ 多为关于 x 的间断函数,流通量关于空间依赖,本质上也是关于空间间断.

流通量间断问题存在于多孔介质渗流、水波、弹性波和交通流等问题中,应用广泛,已成为近年来研究的热点问题,涌现出不少研究成果[1-16].交通中流通量间断问题主要是由于道路的非均匀性引起的,非均匀道路一般指路面宽度或车道数变化、交通信号灯、出入匝道口和收费站等路段.目前研究流通量间断交通流模型主要有非均匀道路条件下的推广 LWR 模型[6-8,17-20]和多车种 LWR 模型[13,21-26],此外,Zhang 等[18]基于推广 LWR 模型,将 Zhang 模型[27]进行推广,获得了一个可变车道数和自由流速度的高阶交通流模型,称为推广 Zhang 模型.

对于流通量间断交通流模型,因其流通量函数关于空间间断的本质,经典的一阶单调格式及非线性 WENO(weighted essentially non-oscillatory)和 RKDG(Runge-Kutta discontinous Galer-

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kin）高阶数值格式难以直接应用[6-8,22].Zhang 和 Liu[6,8]讨论了方程（1）的齐次标量形式，通过对其特征线的研究，提出了将依赖于空间变化的参数 $\theta(x)$"凝固"在网格边界的 δ 映射算法，进而运用经典的 Godunov、EO 和 LF 等数值流通量，构造了流通量间断交通流模型的一阶数值格式，进一步地，将 δ 映射算法分别与 WENO 和 RKDG 方法结合，设计了流通量间断交流通模型的高阶 WENO 和 RKDG 格式，详见文献［7，9，16，23-25］.另外，Zhang 等[22]视 $\theta(x)$ 为解变量，将流通量间断多车种 LWR 模型标准化为双曲守恒律方程，运用 WENO 格式求解.此外，Bale 和 Leveque[2]基于双曲守恒律和平衡律方程，将波传播算法与有限体积法结合，提出了一种求解方程（1）的高分辨率有限体积法.Bürger 等[11,19,21]深入研究了流通量间断的运动学模型，并给出一组差分格式集.Jin 等[10]引入道路供求基本图，并在供求空间中构建了求解推广 LWR 模型 Riemann 解的新框架.Wang[12]基于流通量间断双曲守恒律的标量形式，通过引入推广的 EO 流通量函数，提出了推广的 EO 单调差分格式.Chen 等[13]提出了一种高分辨率松弛格式，用于求解流通量间断多车种 LWR 模型.Wiens 等[14]运用柔化函数，导出了含单个间断点的分段线性流通量双曲守恒律 Riemann 问题的解析解.

尽管针对流通量间断交通流模型已有不少的研究工作，但多集中于研究描述平衡态交通流的推广 LWR 模型和多车种 LWR 模型，对非均匀道路条件下各向异性高阶交通流模型（简称各向异性模型）研究较少.而各向异性模型在交通流研究中有不可替代的优势，其可以描述平衡态与非平衡态交通流，无论在路段还是路网上，均能够很好地刻画实际交通现象.正因如此，各向异性模型一直受到很大的重视，对其数值算法的研究亦有不少工作：Lebacque 等[28-30]利用供求思想，讨论了二阶交通流模型的 Riemann 解，并运用各向异性高阶交通流模型 Riemann 不变量的性质，证明了 GSOM（generic second order traffic flow models）的解析解等价于 LWR 模型的分段解.Mammar 等[31]给出 ARZ（Aw-Rascle-Zhang）模型 Riemann 解和 Godunov 格式.Zhang 等[32]运用 CHO 模型 Riemann 不变量性质，获得其 Riemann 解（Godunov 流通量），进一步推广获得了一系列近似 Riemann 解（EO 和 LF 等数值流通量）及相应的一阶数值格式[33].此外，Qiao 等[34]将上述各数值流通量分别与 WENO 和 RKDG 方法结合，获得求解 CHO 模型的高阶 WENO 和 RKDG 格式.

本文在上述研究工作的基础上，将各向异性 CHO 模型进行推广，获得非均匀道路条件下的流通量间断 CHO 模型，并基于其 Riemann 不变量性质，运用局部简化方法及 δ 映射算法，设计其一阶数值格式.第 1 节给出了流通量间断 CHO 模型；第 2 节设计了流通量间断 CHO 模型的一阶数值格式；第 3 节模拟了流通量间断 CHO 模型；第 4 节总结了全文.

1　流通量间断 CHO 模型及其特征结构

1.1　流通量间断 CHO 模型

Zhang 等[32]在交通中引入伪密度"w"，提出 CHO 模型：

$$\rho_t + (\rho V(w))_x = 0, \tag{2}$$

$$w_t + (wV(w))_x = \frac{v_e(\rho) - V(w)}{-\tau V'(w)}, \tag{3}$$

其中 $\rho = \rho(x,t)$ 和 $w = w(x,t)$ 分别表示位置 x 处、t 时刻的密度和伪密度，伪密度 w 由已知函数 $v = V(w)$ 转换得到，v 为车流速度，$V(w)$ 满足一般速度-密度关系的基本性质，即 $V'(w) < 0$，$0 \leqslant w \leqslant \rho_{jam}$（$\rho_{jam}$ 为阻塞密度），$v_e(\rho)$ 为平衡速度-密度关系，τ 为松弛时间.若 $\rho = w$，$V(w) = v_e(\rho)$，则 CHO 模型（2）和（3）退化为 LWR 模型[35-36]，表明 CHO 模型与 LWR 模型是相容的.

Zhang 等[18]给出可变车道数和自由流速度的 LWR 模型：

$$(a\rho)_t + (a\rho v_e(\rho,b))_x = 0, \tag{4}$$

其中 $a = a(x)$ 为位置 x 处的车道数，$b = b(x) = v_m(x)/\max_x\{v_m(x)\}$，$v_m(x)$ 是位置 x 处的自由流速度．称方程(4)为推广 LWR 模型(ELWR 模型)．

结合文献[32]中运用 LWR 模型的加速度方程导出 CHO 模型中“动量”守恒方程(3)的思想，本文基于 ELWR 模型(4)及其加速度方程，导出可变车道数和自由流速度的 CHO 模型：

$$(a\rho)_t + (a\rho V(w,b))_x = 0, \tag{5}$$

$$(aw)_t + (awV(w,b))_x = \frac{a(V(w,b) - v_e(\rho,b))}{-\tau(\partial V(w,b)/\partial w)}, \tag{6}$$

其中 $\partial V(w,b)/\partial w$ 为速度关于伪密度的偏导数，且 $\partial V(w,b)/\partial w < 0$．称方程组(5)和(6)为流通量间断 CHO 模型，简称为推广 CHO 模型(ECHO 模型)，方程(5)和(6)分别为 ECHO 模型的质量守恒方程和“动量”守恒方程．若 $w=\rho$，$V(w,b)=v_e(\rho,b)$，则 ECHO 模型退化为 ELWR 模型，表明 ECHO 模型与 ELWR 模型是相容的．

1.2 流通量间断 CHO 模型 Riemann 问题及其特征结构

记 $a\rho \triangleq U$，$aw \triangleq W$，则 ECHO 模型可写为

$$U_t + (UV(W/a,b))_x = 0, \tag{7}$$

$$W_t + (WV(W/a,b))_x = \frac{a(V(W/a,b) - v_e(U/a,b))}{-\tau(\partial V(W/a,b)/\partial(W/a))}. \tag{8}$$

若取

$$\boldsymbol{u} = (U,W)^{\mathrm{T}},\ \boldsymbol{\theta}(x) = (a,b)^{\mathrm{T}},$$

$$\boldsymbol{f}(\boldsymbol{u},\boldsymbol{\theta}(x)) = (f_1, f_2)^{\mathrm{T}} = (f_1(U,W,a,b), f_2(W,a,b))^{\mathrm{T}} = (UV(W/a,b), WV(W/a,b))^{\mathrm{T}},$$

$$\boldsymbol{s}(\boldsymbol{u},\boldsymbol{\theta}(x)) = (s_1,s_2)^{\mathrm{T}} = (s_1(U,W,a,b), s_2(U,W,a,b))^{\mathrm{T}} = \left(0, \frac{a(V(W/a,b) - v_e(U/a,b))}{-\tau(\partial V(W/a,b)/\partial(W/a))}\right)^{\mathrm{T}},$$

则 ECHO 模型统一为流通量间断双曲守恒律方程(1)．其分量形式为

$$\begin{cases} U_t + f_1(U,W,a,b)_x = 0, \\ W_t + f_2(W,a,b)_x = s_2(U,W,a,b), \end{cases} \tag{9}$$

且满足

$$f_2 = Zf_1,\ f_1 = \frac{1}{Z}f_2, \tag{10}$$

其中 $Z = W/U > 1$，反映道路上伪密度 $W(=aw)$ 与实际密度 $U(=a\rho)$ 的偏差．

固定 a 和 b，ECHO 模型的 Jacobi 矩阵为

$$\boldsymbol{f}_{\boldsymbol{u}} = \begin{pmatrix} f_{1U} & f_{1W} \\ f_{2U} & f_{2W} \end{pmatrix} = \begin{pmatrix} V(W/a,b) & \dfrac{U}{a}\dfrac{\partial V(W/a,b)}{\partial(W/a)} \\ 0 & V(W/a,b) + \dfrac{W}{a}\dfrac{\partial V(W/a,b)}{\partial(W/a)} \end{pmatrix},$$

其对应的特征值为

$$\lambda_1 = \lambda_1(U,W,a,b) = V(W/a,b) + \frac{W}{a}\frac{\partial V(W/a,b)}{\partial(W/a)},$$

$$\lambda_2 = \lambda_2(U, W, a, b) = V(W/a, b),$$

且 $\lambda_1 \leqslant \lambda_2 = V(W/a, b)$，表明 ECHO 模型是各向异性模型．

在 ECHO 模型(9)中，令 $s_2(U, W, a, b) = 0$，得其齐次形式：

$$\begin{cases} U_t + f_1(U, W, a, b)_x = 0, \\ W_t + f_2(W, a, b)_x = 0. \end{cases} \tag{11}$$

若齐次形式的初始条件为

$$\boldsymbol{u}_0(x) = \boldsymbol{u}(x, 0) = \begin{cases} \boldsymbol{u}_1, & x < 0, \\ \boldsymbol{u}_2, & x > 0, \end{cases} \quad \boldsymbol{\theta}(x) = \begin{cases} \boldsymbol{\theta}_1, & x < 0, \\ \boldsymbol{\theta}_2, & x > 0, \end{cases} \tag{12}$$

其中

$$\boldsymbol{u}(x, 0) = (U(x, 0), W(x, 0))^{\mathrm{T}}, \ \boldsymbol{u}_1 = (U_1, W_1)^{\mathrm{T}}, \ \boldsymbol{u}_2 = (U_2, W_2)^{\mathrm{T}},$$

$$\boldsymbol{\theta}(x) = (a(x), b(x))^{\mathrm{T}}, \ \boldsymbol{\theta}_1 = (a_1, b_1)^{\mathrm{T}}, \ \boldsymbol{\theta}_2 = (a_2, b_2)^{\mathrm{T}},$$

则称初值问题(11)和(12)为 Riemann 问题．

对方程(8)两端同乘 $(WV(W/a, b))_W$ 得

$$(WV(W/a, b))_W W_t + (WV(W/a, b))_W (WV(W/a, b))_x = (WV(W/a, b))_W s_2(U, W, a, b).$$

因

$$(WV(W/a, b))_W W_t = (WV(W/a, b))_t, \ (WV(W/a, b))_W = \lambda_1,$$

则得 ECHO 模型的一特征方程为

$$(WV(W/a, b))_t + \lambda_1 (WV(W/a, b))_x = (WV(W/a, b))_W s_2(U, W, a, b). \tag{13}$$

注意到方程(8)可以写成输运形式：

$$(UZ)_t + (UZV(UZ/a, b))_x = s_2(U, UZ, a, b). \tag{14}$$

将方程(7)代入方程(14)，得另一特征方程为

$$Z_t + \lambda_2 Z_x = \frac{s_2(U, UZ, a, b)}{U}. \tag{15}$$

由特征方程(13)和(15)产生的特征场分别称为“1-特征场”和“2-特征场”：

① 1-特征场　特征值 λ_1，特征变量 $WV(W/a, b)$；

② 2-特征场　特征值 λ_2，特征变量 Z．

根据特征线理论，特征变量 Z 是 1-特征场的 Riemann 不变量[37]．特征值 λ_1 和 λ_2 的右特征向量分别为 $\boldsymbol{R}_1 = (U, W)^{\mathrm{T}}$ 和 $\boldsymbol{R}_2 = (1, 0)^{\mathrm{T}}$，且

$$\nabla\lambda_1 \cdot \boldsymbol{R}_1 = \left(\frac{\partial\lambda_1}{\partial U}, \frac{\partial\lambda_1}{\partial W}\right)(U, W)^{\mathrm{T}} = \frac{W}{a}\left(2\frac{\partial V(W/a, b)}{\partial(W/a)} + \frac{W}{a}\frac{\partial^2 V(W/a, b)}{\partial(W/a)^2}\right) \neq 0,$$

$$\nabla\lambda_2 \cdot \boldsymbol{R}_2 = \left(\frac{\partial\lambda_2}{\partial U}, \frac{\partial\lambda_2}{\partial W}\right)(1, 0)^{\mathrm{T}} = 0.$$

因 $\nabla\lambda_1 \cdot \boldsymbol{R}_1 \neq 0$，则 1- 特征场为非线性特征场，对应激波或稀疏波；$\nabla\lambda_2 \cdot \boldsymbol{R}_2 = 0$，则 2-特征场为线性退化特征场，对应接触间断[38]．又因方程(14)是输运形式，Z 沿特征线穿过 1-特征场保持不变．

2　一阶数值格式

因 ECHO 模型是流通量间断双曲守恒律方程，可直接运用 Zhang 等提出的 δ 映射算法[6,8]求解，对应的一阶数值格式称为格式 1．又因 ECHO 模型是各向异性交通流模型，利用其 Riemann 不变量性质,分别通过 $W = Z^- U$ 和 $U = W/Z^-$ 局部简化 ECHO 模型,获得流通量间断的简化模型 1 和 2(Z^- 表示 Z 在界面上游的值),再运用δ映射算法求解简化模型 1 和 2,对应的一阶数值格式称为格式 2 和 3.以下将逐一介绍格式 1~3.

给定空间计算区间 $[0,L]$ 的均匀划分 $\{I_i\}_{i=1}^{N}$, $I_i = (x_{i-1/2}, x_{i+1/2})$, $\Delta_i = x_{i+1/2} - x_{i-1/2}$, $x_i = (x_{i-1/2} + x_{i+1/2})/2$, $i = 1,2,\cdots,N$, 使集合 $\{I_i\}_{i=1}^{N}$ 正好覆盖计算区间,且根据需要在边界外添加虚拟网格(如图 1),并给定时间计算区间 $[0,T]$ 的划分 $\{\Delta t^n\}_{n=0}^{M}$, $\Delta t^n = t^{n+1} - t^n (n = 0,1,\cdots, M-1)$.取

$$\boldsymbol{u}_i^n = \boldsymbol{u}(x_i, t^n),\ \boldsymbol{\theta}_i = \boldsymbol{\theta}(x_i), \tag{16}$$

或

$$\boldsymbol{u}_i^n = \frac{1}{\Delta_i}\int_{I_i} \boldsymbol{u}(\xi, t^n)\,\mathrm{d}\xi,\ \boldsymbol{\theta}_i = \frac{1}{\Delta_i}\int_{I_i} \boldsymbol{\theta}(\xi)\,\mathrm{d}\xi. \tag{17}$$

式(16)和(17)分别对应有限差分和有限体格式.

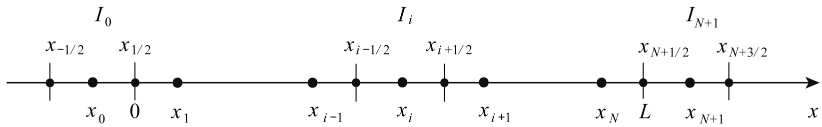


图 1 空间网格划分

Fig. 1 Cell division for space discretization

2.1 格式 1

选取一个介于 $\boldsymbol{\theta}_i$ 和 $\boldsymbol{\theta}_{i+1}$ 的中间状态 $\boldsymbol{\theta}_{i+1/2}$, $\boldsymbol{\theta}_{i+1/2} = \bar{\boldsymbol{\theta}}(\boldsymbol{\theta}_i, \boldsymbol{\theta}_{i+1})$, 其中 $\bar{\boldsymbol{\theta}}$ 为连续函数,且满足相容性 $\bar{\boldsymbol{\theta}}(\boldsymbol{\theta}, \boldsymbol{\theta}) = \boldsymbol{\theta}$.若流通量间断双曲守恒律方程(1) 为标量方程,$\theta_{i+1/2}$ 常取为[8]

$$\theta_{i+1/2} = \begin{cases} \theta_i, & f(u_i^*, \theta_i) < f(u_{i+1}^*, \theta_{i+1}), \\ \theta_{i+1}, & \text{otherwise}, \end{cases}$$

其中 $f(u_i^*, \theta_i)$ 表示 θ_i 状态的最大流量值.

对 U_j^n, $W_j^n (j = i, i+1)$, $\delta_{i+1/2}$ 映射定义为:取 $\gamma \in (-\infty, 1]$ 最大,使

$$f_1(\delta_{i+1/2} U_j^n, \delta_{i+1/2} W_j^n, a_{i+1/2}, b_{i+1/2}) = \gamma f_1(U_j^n, W_j^n, a_j, b_j), \tag{18}$$

$$f_2(\delta_{i+1/2} W_j^n, a_{i+1/2}, b_{i+1/2}) = \gamma f_2(W_j^n, a_j, b_j), \tag{19}$$

$$\lambda_l(\delta_{i+1/2} U_j^n, \delta_{i+1/2} W_j^n, a_{i+1/2}, b_{i+1/2}) \lambda_l(U_j^n, W_j^n, a_j, b_j) \geqslant 0, \qquad l = 1,2 \tag{20}$$

有解,且

$$\begin{cases} \lambda_l(\delta_{i+1/2} U_i^n, \delta_{i+1/2} W_i^n, a_{i+1/2}, b_{i+1/2}) \geqslant 0, & \lambda_l(U_i^n, W_i^n, a_i, b_i) = 0, \\ \lambda_l(\delta_{i+1/2} U_{i+1}^n, \delta_{i+1/2} W_{i+1}^n, a_{i+1/2}, b_{i+1/2}) \leqslant 0, & \lambda_l(U_{i+1}^n, W_{i+1}^n, a_{i+1}, b_{i+1}) = 0, \end{cases} \tag{21}$$

其中 γ 最大值 $\gamma_{\max} = \gamma_{\max}(U_j^n, W_j^n, a_j, b_j, a_{i+1/2}, b_{i+1/2})$, 依赖于 U_j^n, W_j^n, a_j, b_j, $a_{i+1/2}$ 和 $b_{i+1/2}$.

对于上述的空间和时间网格划分,ECHO 模型(9)的一阶数值格式为

$$\begin{cases} U_i^{n+1} = U_i^n - \dfrac{\Delta t^n}{\Delta_i}(\hat{f}_{1_{i+1/2}} - \hat{f}_{1_{i-1/2}}), \\ W_i^{n+1} = W_i^n - \dfrac{\Delta t^n}{\Delta_i}(\hat{f}_{2_{i+1/2}} - \hat{f}_{2_{i-1/2}}) + \Delta t^n s_2(U_i^n, W_i^n, a_i, b_i), \end{cases} \tag{22}$$

其中

$$\hat{f}_{1_{i+1/2}} = \hat{f}_1(U_i^n, U_{i+1}^n; W_i^n, W_{i+1}^n; a_i, a_{i+1}; b_i, b_{i+1}) =$$

$$\hat{f}_1(\delta_{i+1/2}U_i^n,\delta_{i+1/2}U_{i+1}^n;\delta_{i+1/2}W_i^n,\delta_{i+1/2}W_{i+1}^n;a_{i+1/2};b_{i+1/2}) \triangleq \tilde{f}_{1_{i+1/2}}, \tag{23}$$

$$\hat{f}_{2_{i+1/2}} = \hat{f}_2(W_i^n,W_{i+1}^n;a_i,a_{i+1};b_i,b_{i+1}) = \hat{f}_2(\delta_{i+1/2}W_i^n,\delta_{i+1/2}W_{i+1}^n;a_{i+1/2};b_{i+1/2}) \triangleq \tilde{f}_{2_{i+1/2}}. \tag{24}$$

若模型方程为流通量间断标量守恒方程(如 ELWR 模型),则可采用 Godunov、EO 和 LF 等经典数值流通量[6-7].考虑到 ECHO 模型为方程组, $\tilde{f}_{1_{i+1/2}}$ 和 $\tilde{f}_{2_{i+1/2}}$ 常采用简单适用的 LF 数值流通量:

$$\tilde{f}^{\mathrm{LF}}_{1_{i+1/2}} = \frac{1}{2}[f_1(\delta_{i+1/2}U_i^n,\delta_{i+1/2}W_i^n,a_{i+1/2},b_{i+1/2}) + f_1(\delta_{i+1/2}U_{i+1}^n,\delta_{i+1/2}W_{i+1}^n,a_{i+1/2},b_{i+1/2}) - \alpha^n(\delta_{i+1/2}U_{i+1}^n - \delta_{i+1/2}U_i^n)], \tag{25}$$

$$\tilde{f}^{\mathrm{LF}}_{2_{i+1/2}} = \frac{1}{2}[f_2(\delta_{i+1/2}W_i^n,a_{i+1/2},b_{i+1/2}) + f_2(\delta_{i+1/2}W_{i+1}^n,a_{i+1/2},b_{i+1/2}) - \alpha^n(\delta_{i+1/2}W_{i+1}^n - \delta_{i+1/2}W_i^n)], \tag{26}$$

$$\alpha^n = \max_{i,j}(|\lambda_1(\delta_{i+1/2}U_j^n,\delta_{i+1/2}W_j^n,a_{i+1/2},b_{i+1/2})|, |\lambda_2(\delta_{i+1/2}U_j^n,\delta_{i+1/2}W_j^n,a_{i+1/2},b_{i+1/2})|). \tag{27}$$

若

$$\alpha^n = \max_{j}(|\lambda_1(\delta_{i+1/2}U_j^n,\delta_{i+1/2}W_j^n,a_{i+1/2},b_{i+1/2})|, |\lambda_2(\delta_{i+1/2}U_j^n,\delta_{i+1/2}W_j^n,a_{i+1/2},b_{i+1/2})|),$$

则式(25)和(26)为 ECHO 模型的 LLF(local Lax-Friedrichs)数值流通量.为保证格式的数值稳定性,式(22)的时间步长 Δt^n 应满足 CFL 条件[39]:

$$\Delta t^n = C\Delta_i/\alpha^n, \qquad C \leqslant 1, \tag{28}$$

其中 α^n 如式(27)所示.

2.2 局部简化

注意到 ECHO 模型(9)关于 λ_2 特征速度的迎风性,且 Z 是 1-特征场的 Riemann 不变量,在界面附近假定近似有 $Z = Z^-$, 结合式(10),ECHO 模型局部简化为

$$\begin{cases} U_t + f_1(U,Z^-U,a,b)_x = 0, \\ W_t + [Z^-f_1(U,Z^-U,a,b)]_x = s_2(U,W,a,b), \end{cases} \tag{29}$$

或

$$\begin{cases} U_t + \left[\dfrac{1}{Z^-}f_2(W,a,b)\right]_x = 0, \\ W_t + f_2(W,a,b)_x = s_2(U,W,a,b), \end{cases} \tag{30}$$

称方程组(29)和(30)分别为 ECHO 模型的简化模型 1 和 2.在简化模型 1 和 2 中,令 $s_2(U,W,a,b) = 0$, 得其齐次形式分别为

$$\begin{cases} U_t + f_1(U,Z^-U,a,b)_x = 0, \\ W_t + [Z^-f_1(U,Z^-U,a,b)]_x = 0 \end{cases} \tag{31}$$

和

$$\begin{cases} U_t + \left[\dfrac{1}{Z^-}f_2(W,a,b)\right]_x = 0, \\ W_t + f_2(W,a,b)_x = 0, \end{cases} \tag{32}$$

其中,方程组(31)的第一个方程为关于解变量 U 的流通量间断标量守恒方程,方程组(32)的第二个方程为关于解变量 W 的流通量间断标量守恒方程.

2.3 格式 2

对前述的网格划分, $Z^- = Z_i^n = W_i^n / U_i^n$.基于方程组(31)的第一个方程,对 $U_j^n (j = i, i+1)$, $\delta_{i+1/2}^1$ 映射定义为:取 $\gamma_1 \in (-\infty, 1]$ 最大,使

$$\begin{cases} f_1(\delta_{i+1/2}^1 U_j^n, Z_i^n \delta_{i+1/2}^1 U_j^n, a_{i+1/2}, b_{i+1/2}) = \gamma_1 f_1(U_j^n, Z_i^n U_j^n, a_j, b_j), \\ \lambda(\delta_{i+1/2}^1 U_j^n, Z_i^n, a_{i+1/2}, b_{i+1/2}) \lambda(U_j^n, Z_i^n, a_j, b_j) \geqslant 0 \end{cases} \tag{33}$$

有解,且

$$\begin{cases} \lambda(\delta_{i+1/2}^1 U_i^n, Z_i^n, a_{i+1/2}, b_{i+1/2}) \geqslant 0, & \lambda(U_i^n, Z_i^n, a_i, b_i) = 0, \\ \lambda(\delta_{i+1/2}^1 U_{i+1}^n, Z_i^n, a_{i+1/2}, b_{i+1/2}) \leqslant 0, & \lambda(U_{i+1}^n, Z_i^n, a_{i+1}, b_{i+1}) = 0, \end{cases} \tag{34}$$

其中 γ_1 最大值 $\gamma_{1_{\max}} \triangleq \gamma_{1_{\max}}(U_j^n, Z_i^n, a_j, b_j, a_{i+1/2}, b_{i+1/2})$, 依赖于 U_j^n, Z_i^n, a_j, b_j, $a_{i+1/2}$ 和 $b_{i+1/2}$, $\lambda(U, Z^-, a, b) = \partial f_1(U, Z^- U, a, b) / \partial U$.

方程组(31)在网格边界 $x_{i+1/2}$ 处的数值流通量为

$$\hat{f}_{1_{i+1/2}} = \hat{f}_1(U_i^n, U_{i+1}^n; Z_i^n; a_i, a_{i+1}; b_i, b_{i+1}) = \tilde{f}_1(\delta_{i+1/2}^1 U_i^n, \delta_{i+1/2}^1 U_{i+1}^n; Z_i^n; a_{i+1/2}; b_{i+1/2}) \triangleq \tilde{f}_{1_{i+1/2}}, \tag{35}$$

$$\hat{f}_{2_{i+1/2}} = Z_i^n \hat{f}_{1_{i+1/2}} = Z_i^n \tilde{f}_{1_{i+1/2}}. \tag{36}$$

$\tilde{f}_{1_{i+1/2}}$ 可取任一经典数值流通量,简述如下:

Godunov 流通量

$$\tilde{f}_{1_{i+1/2}}^{G} = \tilde{f}_1^{G}(U_1, U_2; Z_i^n; a_{i+1/2}; b_{i+1/2}) = \begin{cases} \min\limits_{U_1 \leqslant U \leqslant U_2} f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}), & U_1 \leqslant U_2, \\ \max\limits_{U_2 \leqslant U \leqslant U_1} f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}), & U_1 > U_2; \end{cases} \tag{37}$$

EO 数值流通量

$$\tilde{f}_{1_{i+1/2}}^{EO} = \tilde{f}_1^{EO}(U_1, U_2; Z_i^n; a_{i+1/2}; b_{i+1/2}) = \int_0^{U_1} \max(\partial f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}) / \partial U, 0) \mathrm{d}U + \int_0^{U_2} \min(\partial f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}) / \partial U, 0) \mathrm{d}U + f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}) |_{U=0}; \tag{38}$$

LF 数值流通量

$$\tilde{f}_{1_{i+1/2}}^{LF} = \tilde{f}_1^{LF}(U_1, U_2; Z_i^n; a_{i+1/2}; b_{i+1/2}) = \frac{1}{2}[f_1(U_1, Z_i^n U_1, a_{i+1/2}, b_{i+1/2}) + f_1(U_2, Z_i^n U_2, a_{i+1/2}, b_{i+1/2}) - \alpha(U_2 - U_1)],$$
$$\alpha = \max_i \max_{\min(U_1, U_2) \leqslant U \leqslant \max(U_1, U_2)} |\partial f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}) / \partial U|, \tag{39}$$

若 $\alpha = \max_{\min(U_1, U_2) \leqslant U \leqslant \max(U_1, U_2)} |\partial f_1(U, Z_i^n U, a_{i+1/2}, b_{i+1/2}) / \partial U|$, 则式(39)为 LLF 数值流通量.

综上,基于方程组(29),采用式(37)的 Godunov 流通量,可得 ECHO 模型(9)的一阶 Godunov 格式:

$$
\begin{cases}
U_i^{n+1} = U_i^n - \dfrac{\Delta t^n}{\Delta_i}(\tilde{f}^{\mathrm{G}}_{1_{i+1/2}} - \tilde{f}^{\mathrm{G}}_{1_{i-1/2}}), \\
W_i^{n+1} = W_i^n - \dfrac{\Delta t^n}{\Delta_i}(Z_i^n \tilde{f}^{\mathrm{G}}_{1_{i+1/2}} - Z_{i-1}^n \tilde{f}^{\mathrm{G}}_{1_{i-1/2}}) + \Delta t^n s_2(U_i^n, W_i^n, a_i, b_i).
\end{cases} \tag{40}
$$

若将 $\tilde{f}^{\mathrm{G}}_{1_{i+1/2}}$ 分别替换为式(38)和(39)的EO和LF(或LLF)数值流通量,则得ECHO模型的一阶EO和LF(或LLF)格式.同样地,式(40)的 Δt^n 需满足形如式(28)的CFL条件,此时 α^n 应替换为

$$
\alpha^n = \max_{i,j}(|\lambda_1(\delta^1_{i+1/2}U_j^n, Z_i^n\delta^1_{i+1/2}U_j^n, a_{i+1/2}, b_{i+1/2})|,
$$
$$
|\lambda_2(\delta^1_{i+1/2}U_j^n, Z_i^n\delta^1_{i+1/2}U_j^n, a_{i+1/2}, b_{i+1/2})|).
$$

2.4 格式3

基于方程组(32)的第二个方程,对 $W_j^n(j=i,i+1)$, $\delta^2_{i+1/2}$ 映射定义为:取 $\gamma_2 \in (-\infty, 1]$ 最大,使

$$
\begin{cases}
f_2(\delta^2_{i+1/2}W_j^n, a_{i+1/2}, b_{i+1/2}) = \gamma_2 f_2(W_j^n, a_j, b_j), \\
\lambda(\delta^2_{i+1/2}W_j^n, a_{i+1/2}, b_{i+1/2})\lambda(W_j^n, a_j, b_j) \geqslant 0
\end{cases} \tag{41}
$$

有解,且

$$
\begin{cases}
\lambda(\delta^2_{i+1/2}W_i^n, a_{i+1/2}, b_{i+1/2}) \geqslant 0, \qquad \lambda(W_i^n, a_i, b_i) = 0, \\
\lambda(\delta^2_{i+1/2}W_{i+1}^n, a_{i+1/2}, b_{i+1/2}) \leqslant 0, \qquad \lambda(W_{i+1}^n, a_{i+1}, b_{i+1}) = 0,
\end{cases} \tag{42}
$$

其中 γ_2 最大值 $\gamma_{2_{\max}} \triangleq \gamma_{2_{\max}}(W_i^n, a_j, b_j, a_{i+1/2}, b_{i+1/2})$, 依赖于 W_j^n, a_j, b_j, $a_{i+1/2}$ 和 $b_{i+1/2}$, $\lambda(W,a,b) = \partial f_2(W,a,b)/\partial W$.

方程组(32)在网格边界 $x_{i+1/2}$ 处的数值流通量为

$$
\hat{f}_{2_{i+1/2}} = \hat{f}_2(W_i^n, W_{i+1}^n; a_i, a_{i+1}; b_i, b_{i+1}) =
$$
$$
\tilde{f}_2(\delta^2_{i+1/2}W_i^n, \delta^2_{i+1/2}W_{i+1}^n; a_{i+1/2}; b_{i+1/2}) \triangleq \tilde{f}_{2_{i+1/2}}, \tag{43}
$$

$$
\hat{f}_{1_{i+1/2}} = \frac{1}{Z_i^n}\hat{f}_{2_{i+1/2}} = \frac{1}{Z_i^n}\tilde{f}_{2_{i+1/2}}. \tag{44}
$$

$\tilde{f}_{2_{i+1/2}}$ 可取任一经典数值流通量,简述如下:

Godunov 流通量

$$
\tilde{f}^{\mathrm{G}}_{2_{i+1/2}} = \tilde{f}^{\mathrm{G}}_2(W_1, W_2; a_{i+1/2}; b_{i+1/2}) = \begin{cases}
\min\limits_{W_1 \leqslant W \leqslant W_2} f_2(W, a_{i+1/2}, b_{i+1/2}), \qquad W_1 \leqslant W_2, \\
\max\limits_{W_2 \leqslant W \leqslant W_1} f_2(W, a_{i+1/2}, b_{i+1/2}), \qquad W_1 > W_2;
\end{cases} \tag{45}
$$

EO 数值流通量

$$
\tilde{f}^{\mathrm{EO}}_{2_{i+1/2}} = \tilde{f}^{\mathrm{EO}}_2(W_1, W_2; a_{i+1/2}; b_{i+1/2}) =
$$
$$
\int_0^{W_1}\max(\partial f_2(W, a_{i+1/2}, b_{i+1/2})/\partial W, 0)\,\mathrm{d}W +
$$
$$
\int_0^{W_2}\min(\partial f_2(W, a_{i+1/2}, b_{i+1/2})/\partial W, 0)\,\mathrm{d}W + f_1(W, a_{i+1/2}, b_{i+1/2})|_{W=0}; \tag{46}
$$

LF 数值流通量

$$
\tilde{f}^{\mathrm{LF}}_{2_{i+1/2}} = \tilde{f}^{\mathrm{LF}}_2(W_1, W_2; a_{i+1/2}; b_{i+1/2}) =
$$
$$
\frac{1}{2}[f_2(W_1, a_{i+1/2}, b_{i+1/2}) + f_2(W_2, a_{i+1/2}, b_{i+1/2}) - \alpha(W_2 - W_1)],
$$

$$\alpha = \max_i \max_{\min(W_1,W_2)\leqslant W\leqslant \max(W_1,W_2)} |\partial f_2(W,a_{i+1/2},b_{i+1/2})/\partial W|, \tag{47}$$

若 $\alpha = \max_{\min(W_1,W_2)\leqslant W\leqslant \max(W_1,W_2)} |\partial f_2(W,a_{i+1/2},b_{i+1/2})/\partial W|$，则式(47)为 LLF 数值流通量.

综上,基于方程组(30),采用式(45)的 Godunov 流通量,可得 ECHO 模型(9)的一阶 Godunov 格式:

$$\begin{cases} U_i^{n+1} = U_i^n - \dfrac{\Delta t^n}{\Delta_i}\left(\dfrac{1}{Z_i^n}\tilde{f}^{\mathrm{G}}_{2_{i+1/2}} - \dfrac{1}{Z_{i-1}^n}\tilde{f}^{\mathrm{G}}_{2_{i-1/2}}\right), \\ W_i^{n+1} = W_i^n - \dfrac{\Delta t^n}{\Delta_i}(\tilde{f}^{\mathrm{G}}_{2_{i+1/2}} - \tilde{f}^{\mathrm{G}}_{2_{i-1/2}}) + \Delta t^n s_2(U_i^n, W_i^n, a_i, b_i). \end{cases} \tag{48}$$

若将 $\tilde{f}^{\mathrm{G}}_{2_{i+1/2}}$ 分别替换为形如式(46)和(47)的 EO 和 LF(或 LLF)数值流通量,则得 ECHO 模型的一阶 EO 和 LF(或 LLF)格式.此时,CFL 条件式(28)的 α^n 应替换为

$$\alpha^n = \max_{i,j}(|\lambda_1(\delta^2_{i+1/2}W_j^n/Z_i^n, \delta^2_{i+1/2}W_j^n, a_{i+1/2}, b_{i+1/2})|, \\ |\lambda_2(\delta^2_{i+1/2}W_j^n/Z_i^n, \delta^2_{i+1/2}W_j^n, a_{i+1/2}, b_{i+1/2})|).$$

3 数值模拟

算例 1 基于 ELWR 模型和 ECHO 模型齐次形式模拟由车道数变化引起的交通瓶颈现象,本质上是模拟 Riemann 问题.算例 2 基于 ECHO 模型模拟车道数及自由流速度变化的交通瓶颈问题.算例 3 基于 ELWR 和 ECHO 模型模拟交通扰动的演化.实际模拟中涉及的变量及参数均无量纲化.

3.1 算例 1

设道路长度为 L.取 $b(x)=1$，车道数

$$a(x) = \begin{cases} 3, & x < 0.3, \\ 1, & x \geqslant 0.3, \end{cases} \tag{49}$$

单车道初始密度

$$\rho(x,0) = 0.2\rho_{\mathrm{jam}}, \tag{50}$$

其中 ρ_{jam} 为阻塞密度.

在 ELWR 模型中,取平衡速度-密度关系[6]:

$$v_{\mathrm{e}}(\rho,b) = v_{\mathrm{f}} b(1 - \rho/\rho_{\mathrm{jam}}), \tag{51}$$

其中 v_{f} 为畅行速度,初始条件

$$U(x,0) = a\rho(x,0). \tag{52}$$

在 ECHO 模型中,取平衡速度-密度函数[40]

$$v_{\mathrm{e}}(\rho,b) = v_{\mathrm{f}} b((1 + \mathrm{e}^{(\rho/\rho_{\mathrm{jam}} - 0.25)/0.06})^{-1} - 3.72\times 10^{-6}), \tag{53}$$

速度-密度函数[33-34]

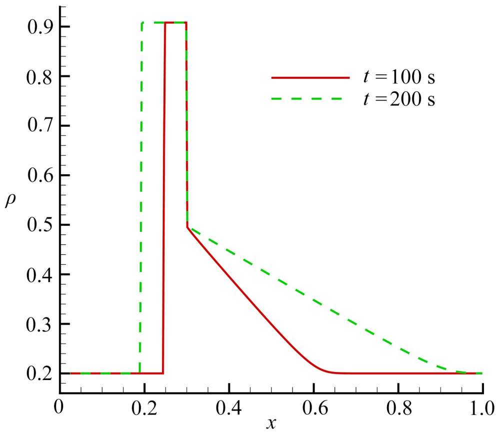
$$V(\rho,b) = v_{\mathrm{f}} b \frac{1 - \rho/\rho_{\mathrm{jam}}}{1 + d(\rho/\rho_{\mathrm{jam}}) + c(\rho/\rho_{\mathrm{jam}})^2}, \qquad c = 4.0,\ d = -0.8, \tag{54}$$

初始条件

$$\begin{cases} U(x,0) = a\rho(x,0), \\ V(W(x,0)/a, b) = v_{\mathrm{e}}(\rho(x,0), b). \end{cases} \tag{55}$$

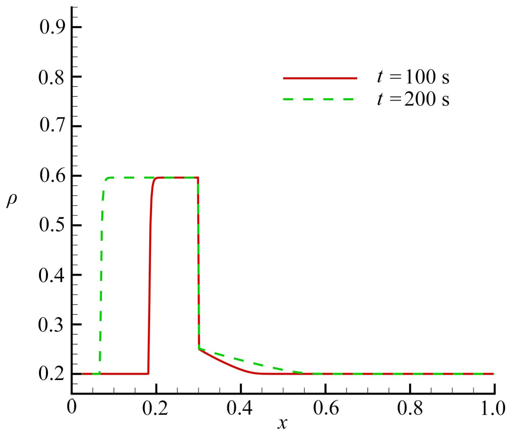
采用一阶外推边界条件,相关参数 $L = 4\,000$ m, $v_{\mathrm{f}} = 20$ m/s, $\Delta_i = 10$ m, $\Delta t^n = 0.2$ s.分别运

用格式 1 和 2 中的一阶 Godunov 格式模拟 ELWR 和 ECHO 模型,模拟结果如图 2 所示.



（a）ELWR 模型

（a）The ELWR model



（b）ECHO 模型

（b）The ECHO model

图 2　车道数由 3 变 1 的交通流密度

Fig. 2　Traffic flow densities for the lane number switching from 3 to 1

图 2 很好地捕捉了由 ELWR 模型和 ECHO 模型齐次形式 Riemann 问题所刻画的交通激波和稀疏波,且在车道数由 3 变 1 的间断处（$x = 0.3$）上游形成向后传播的激波,下游形成向前传播的稀疏波,与实际交通瓶颈的上游形成堵塞,下游逐步疏散的情况吻合.对比图 2(a)和(b)发现,ECHO 模型与 ELWR 模型模拟图像形状相似,进而表明在非均匀道路条件下,由 CHO 模型推广所得的 ECHO 模型是合理有效的.

3.2　算例 2

取车道数

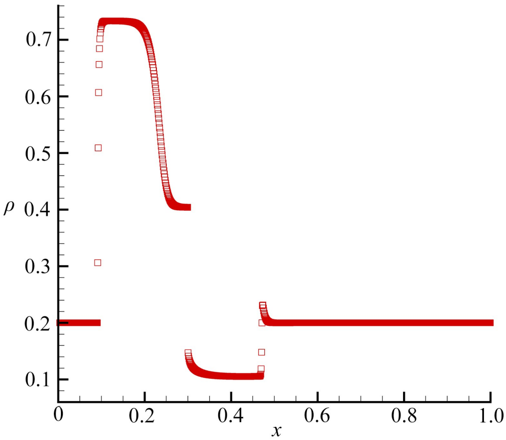
$$a(x) = \begin{cases} 4, & x < 0.3, \\ 2, & x \geqslant 0.3, \end{cases} \tag{56}$$

及

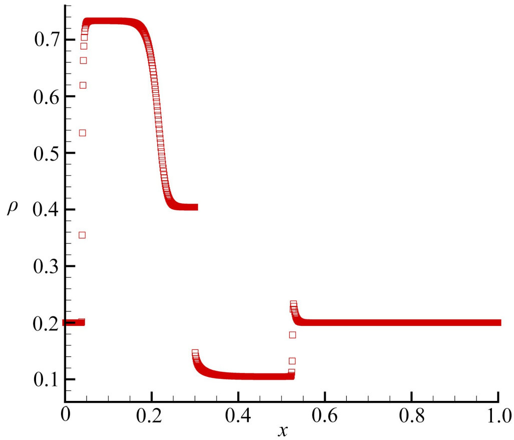
$$b(x) = \begin{cases} 1.0, & x < 0.3, \\ 0.6, & x \geqslant 0.3. \end{cases} \tag{57}$$

平衡速度-密度函数、速度-密度函数和初始条件分别见式(53)~(55),采用一阶外推边界条件,相关参数 $L = 10\,000$ m, $v_{\mathrm{f}} = 20$ m/s, $\tau = 30$ s, $\Delta_i = 10$ m, $\Delta t^n = 0.2$ s.模拟结果如图 3 和图 4 所示.

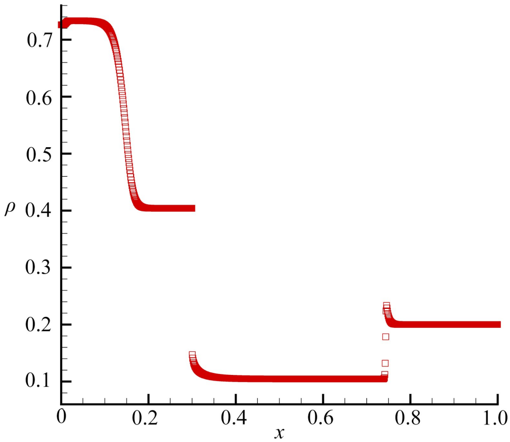
运用格式 2 中的一阶 Godunov 格式模拟 ECHO 模型的结果如图 3 所示,它反映了车道数减少及自由流速度降低路段上交通流密度的变化过程.图 3(a)和(b)显示在间断处（$x = 0.3$）上游形成向后传播的时走时停波(宽幅移动阻塞),且随着时间增加宽幅移动阻塞继续向后传播,如图 3(b)和(c),直至 $t \approx 1\,950$ s 时形成如图 3(d)所示的驻波,与实际交通比较吻合.图 4 为运用一阶 Godunov、EO 和 LLF 格式模拟 ECHO 模型在 $t = 600$ s 的密度对比图,图 4(a)和(b)分别对应格式 2 和 3.由图 4(a)或(b)知:无论是格式 2 或 3,相应的一阶 Godunov、EO 和 LLF 格式模拟的结果差别不大,但数值误差依次增大,与理论分析相符,进而说明利用 ECHO 模型 Riemann 不变量设计的一阶数值格式 2 和 3 是合理有效的.



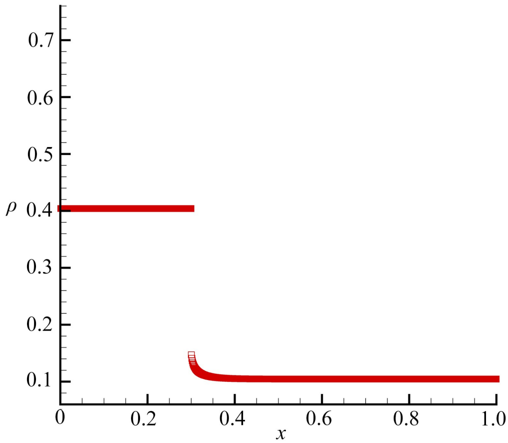
(a) t = 400 s



(b) t = 500 s



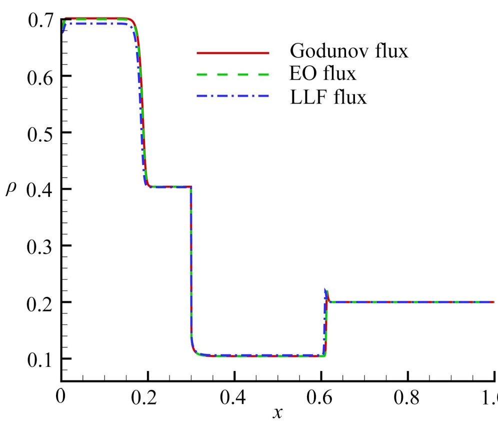
(c) t = 900 s



(d) t = 1 950 s

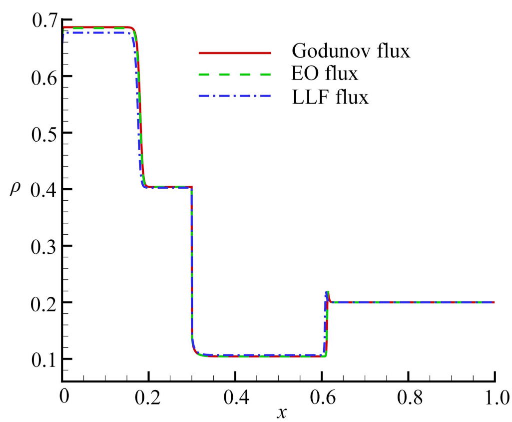
图 3 交通瓶颈在不同时刻的密度变化(ECHO 模型)

Fig. 3 Density changes with time at a bottleneck (ECHO model)



(a) 格式 2

(a) Scheme 2



(b) 格式 3

(b) Scheme 3

图 4 交通瓶颈在 t = 600 s 的密度对比图(ECHO 模型)

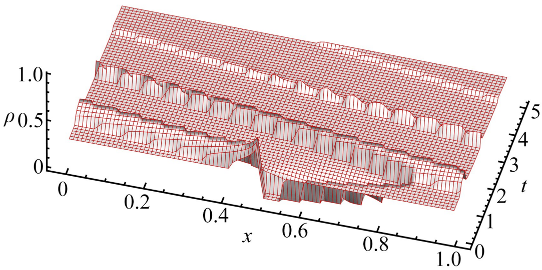
Fig. 4 Densities from different schemes at a bottleneck for t = 600 s (ECHO model)

3.3　算例 3

取车道数 $a(x)$ 为常数, $b(x)$ 为

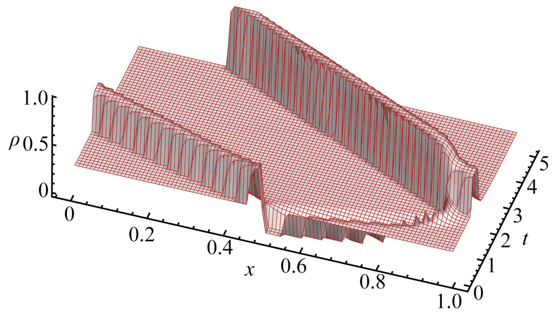
$$b(x)=\begin{cases}10^{-7}, & 0.49<x<0.5,\ 0\ \mathrm{s}\leqslant t\leqslant 30\ \mathrm{s},\\ 1.0, & \text{otherwise}.\end{cases}\tag{58}$$

扰动条件 $b(x)$ 可视为道路上突然发生交通事故，或是交通信号灯为红灯，车辆不能通行. 平衡速度-密度函数和速度-密度函数分别见式(53)和(54)，初始条件如式(55)，其中 $\rho(x,0)=0.25\rho_{\mathrm{jam}}$. 采用周期边界条件，相关参数 $L=2\ 000\ \mathrm{m}$, $v_{\mathrm{f}}=20\ \mathrm{m/s}$, $\tau=10\ \mathrm{s}$, $\Delta_i=2\ \mathrm{m}$, $\Delta t^n=0.04\ \mathrm{s}$. 模拟结果如图 5 和图 6 所示.



（a）ELWR 模型（格式 1）

（a）The ELWR model (scheme 1)

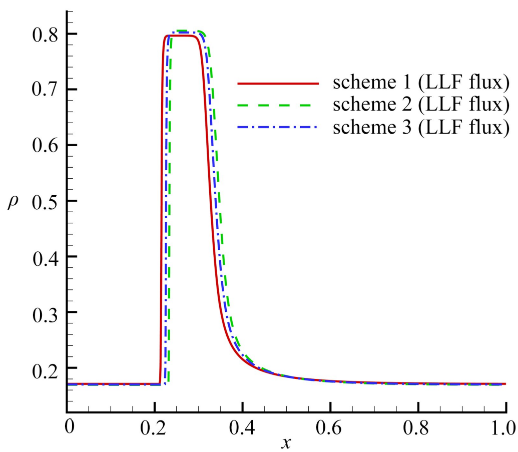


（b）ECHO 模型（格式 3）

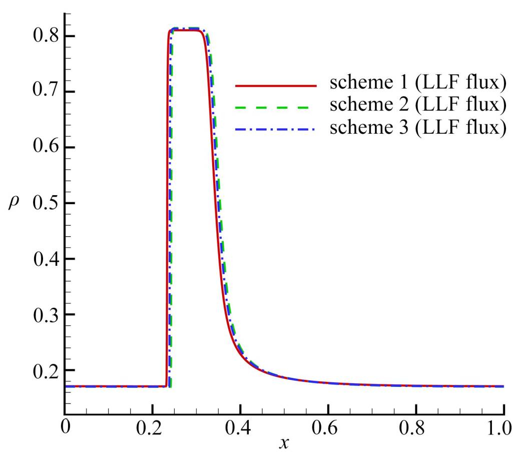
（b）The ECHO model (scheme 3)

图 5　密度随时间的演化（$0\ \mathrm{s}\leqslant t\leqslant 500\ \mathrm{s}$，一阶 Godunov 格式）

Fig. 5　Evolution of density with time ($0\ \mathrm{s}\leqslant t\leqslant 500\ \mathrm{s}$, the first-order Godunov scheme)



（a）$\Delta_i=2\ \mathrm{m}$, $\Delta t^n=0.04\ \mathrm{s}$



（b）$\Delta_i=1\ \mathrm{m}$, $\Delta t^n=0.02\ \mathrm{s}$

图 6　ECHO 模型在不同格式下的密度对比图（$t=500\ \mathrm{s}$）

Fig. 6　Densities from the ECHO model with different schemes ($t=500\ \mathrm{s}$)

图 5(a)和(b)分别显示了 ELWR 和 ECHO 模型在初始扰动条件下交通流密度随时间的演化情况. 图 5(a)显示初始扰动导致交通在间断处上游形成高密度区域，但随着时间的增加逐步消失，最终回到平衡态. 图 5(b)显示初始扰动同样导致交通在间断处上游形成高密度区域，且随着时间的增加逐步演化为向后传播的时走时停波，表明 ECHO 模型能够刻画非平衡态交通流，且再现了与实际交通运行更加吻合的时走时停波现象. 结合算例 1 和 2，进一步表明 ECHO 模型既可以描述平衡态交通流，也可以描述非平衡态交通流，比仅能描述平衡态交通流

的 ELWR 模型更有优势.

图 6 为运用不同的一阶 LLF 格式模拟 ECHO 模型在 $t = 500$ s 的密度对比图.对比图 6(a)和(b)不难发现,随着网格加密,3 种格式模拟结果几乎无甚差别,如图 6(b);但在相对较粗网格下,3 种格式模拟结果有一定差别,如图 6(a).相对于格式 1,格式 2 和 3 的误差较小,进而表明格式 2 和 3 的逼近效果更好.此外,表 1 列出了运用不同一阶 LLF 格式模拟的时间,通过比较表明,格式 2 和 3 的计算效率比格式 1 更高.

表 1 不同格式的运行时间(ECHO 模型)

Table 1 Computation times with different schemes (ECHO model)

space step, time step	scheme 1 (LLF flux)	scheme 2 (LLF flux)	scheme 3 (LLF flux)
$\Delta_i = 2$ m, $\Delta t^n = 0.04$ s	111.77 s	70.64 s	55.39 s
$\Delta_i = 1$ m, $\Delta t^n = 0.02$ s	483.40 s	275.28 s	220.94 s

4 结 论

本文基于 ELWR 模型及其加速度方程导出可变车道数和自由流速度的 ECHO 模型,通过数值模拟验证了由 CHO 模型推广获得的 ECHO 模型是合理有效的,能够描述平衡态和非平衡态交通流,更好地反映了实际交通现象.同时,研究了求解 ECHO 模型的一阶数值格式.一方面,由于 ECHO 模型为流通间断双曲守恒律方程,运用 δ 映射算法设计得到一阶数值格式 1,该格式可应用于求解一般的流通间断双曲守恒律方程,对于流通量间断的标量方程(如 ELWR 模型),可设计经典的一阶 Godunov、EO 和 LF(或 LLF)格式,但对于矢量方程(或方程组),因其流通量函数多为解变量的多元函数(如 ECHO 模型),难以设计误差较小的一阶 Godunov 和 EO 格式,一般采用简单适用的一阶 LF(或 LLF)格式,只是误差相对较大.另一方面,又因 ECHO 模型为流通间断的各向异性交通流模型,利用其 Riemann 不变量性质,运用局部简化方法及 δ 映射算法,设计了求解 ECHO 模型的一阶数值格式 2 和 3,且运用不同的一阶 LLF 格式模拟 ECHO 模型,格式 2 和 3 的逼近效果比格式 1 更好,计算效率也更高,但格式 2 和 3 仅适用于求解流通量间断的各向异性交通流模型.进一步的工作可考虑将设计 ECHO 模型一阶数值格式的思想与高阶数值方法结合,设计 ECHO 模型的高阶数值格式;还可考虑在非均匀道路条件下推广其他各向异性交通流模型及其数值格式.

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A Conserved High-Order Traffic Flow Model With Discontinuous Flux and Its Numerical Simulation

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Abstract: Under inhomogeneous road conditions, a conserved high-order (CHO) anisotropic traffic flow model was extended to obtain a CHO model with discontinuous fluxes. Based on the property of the Riemann invariant for the CHO model with discontinuous fluxes, the first-order Godunov, EO (Engquist-Osher) and LF (Lax-Friedrichs) numerical schemes for this model were designed with the local simplification method and the δ mapping algorithm. The numerical simulations show that, the CHO model with discontinuous fluxes is reasonable and effective. It can describe equilibrium and non-equilibrium traffic flows, and can better describe the actual traffic phenomena compared with the LWR (Lighthill-Whitham-Richards) model with a discontinuous flux.

Key words: CHO model with discontinuous fluxes; Riemann invariant; local simplification; δ mapping

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