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约束 Hamilton 系统的 Lie 对称性及其在场论中的应用*

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摘要: 研究了约束 Hamilton 系统的 Lie 对称性,得到了场论系统的守恒量.首先给出约束 Hamilton 系统的正则运动方程和固有约束方程;其次构建了约束 Hamilton 系统的 Lie 对称性确定方程和结构方程;然后给出了约束 Hamilton 系统的 Lie 守恒定理和守恒量;最后研究了复标量场与 Chern-Simons 项耦合系统的 Lie 对称性和另外一个例子以说明此方法在场论中的应用.

关 键 词: Lie 对称性; 约束 Hamilton 系统; 场论; 守恒量

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引 言

自然界中的四种基本相互作用(QED、QFD、QCD、GR)都是由位形空间中奇异 Lagrange 量描述的,然而过渡到相形空间描述时,可归结为约束 Hamilton 系统[1-3],也就是说正则变量之间存在着约束关系.因此,在对系统进行正则量子化的过程中,如何处理这些约束就相当重要,现已成为人们关注的焦点.并且,对于约束 Hamilton 系统对称性的研究具有广泛的应用,一个重要的应用领域是量子场论(quantum field theory,QFT)[4-7],量子场论是量子力学和经典场论相结合的物理理论,已被广泛应用于粒子物理学和凝聚态物理学中.量子场论的实效理论应用也与 2013 年诺贝尔物理学奖“Higgs 粒子场”的微观量子粒子有着密切的关联.可见,约束 Hamilton 系统在现代物理学中,特别是量子场论中占有十分重要的地位.

一个动力学系统可以用位形空间的 Lagrange 体制和相空间的 Hamilton 体制两种形式描述.而且,Hamilton 力学是经典物理到近代物理的桥梁,它在近代物理和量子理论中有更重要的作用.一般物理运动体系都会受到约束条件的限制,而约束可分为两类,一类是外界施加的约束;另一类是系统的固有约束.前者是由正规 Lagrange 量描述的系统,并且受到外界的干扰力以限制系统位置的完整约束和限制系统速度的非完整约束;后者是由奇异 Lagrange 量描述的系统,在转换到相空间描述时,正则变量之间存在着关系,称为固有约束,也就是说在位形空间中变量间不存在着约束,但采用相空间描述时,正则变量间却存在着约束关系,例如旋转场

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Lagrange 量描述的物理系统.一般来说,由 Lagrange 量描述的物理系统过渡到相空间描述,当相应的 Hesse 矩阵为退化时,其 Lagrange 函数称为奇异 Lagrange 函数,具有奇异 Lagrange 函数的系统称为奇异系统.当奇异系统由 Lagrange 描述过渡到 Hamilton 描述时,需要引入广义动量 p,并进一步将广义速度表示为广义坐标 q 和广义动量 p 的函数,由于 Hesse 矩阵的行列式为零,我们就不能用 q,p 来表示所有的广义动量,因此,在研究系统的对称性时就十分困难.

我们知道,寻求力学系统守恒量的方法主要有 Noether 对称性理论[8-23]、Lie 对称性理论[24-30]和 Mei 对称性理论[31-36],这些方法在国内外已经取得了相当可观的发展[37-44].近年来,李子平等[45-48]研究了约束 Hamilton 系统的 Noether 对称性,并且将这种方法应用在场论中,给出了电磁场、非 Abel 规范场、声子场、电子场和电磁场的相互作用、非 Abel 场与荷电 Bose 场耦合等的 Noether 守恒量.Zhang 和 Tang 等[49-51]研究了场论中导心系统的正则化.然而约束 Hamilton 系统的 Lie 对称性还没有被研究过,以及场论系统的 Lie 确定方程、限制方程、附加限制方程、结构方程都没有给出.Lie 对称性是一个系统的内在性质,为找到 Lie 对称性,需要解复杂的确定方程,再建立结构方程并求规范函数,最后导出相应的守恒量.

本文基于 Lie 对称性思想研究约束 Hamilton 系统的对称性及守恒定理,根据限制方程和附加限制方程给出了场论的强 Lie 对称性和弱 Lie 对称性,给出了相应的强 Lie 守恒量和弱 Lie 守恒量.本文可简单归结如下:第 1 节讨论了约束 Hamilton 系统的内在约束以及其正则方程;第 2 节给出了场论系统的无限小变换和 Lie 对称性理论;第 3 节构造了场论系统的结构方程和守恒定理;第 4 节给出了两个例子,证明其在场论中的应用;第 5 节对文章做了总结以及基于这种方法对未来工作的计划.

1 约束 Hamilton 系统的固有约束及其正则 Hamilton 方程

1.1 约束 Hamilton 系统的固有约束

考虑有限维自由度的系统,场系统的 Lagrange 量密度(不显含时间)为

$$\mathscr{L}=(\varphi^{\alpha},\dot{\varphi}^{\alpha}),\qquad \alpha=1,2,\cdots,n.$$

场系统的 Lagrange 量为

$$L[\varphi^{\alpha},\dot{\varphi}^{\alpha}]=\int_V \mathscr{L}(\varphi^{\alpha}(x),\dot{\varphi}^{\alpha}(x))\,\mathrm{d}^3x.$$

由 Legendre 变换引入正则动量

$$\pi_a=\frac{\delta L}{\delta\dot{\varphi}^a}=\frac{\partial\mathscr{L}}{\partial\dot{\varphi}^a},\tag{1}$$

引入 Hesse 矩阵

$$H_{\alpha\beta}(\varphi,\dot{\varphi})=\frac{\partial^2 L}{\partial\varphi^{\alpha}\partial\dot{\varphi}^{\alpha}},\qquad \alpha,\beta=1,2,\cdots,n.\tag{2}$$

系统的 Euler-Lagrange 方程可表示为

$$H_{\alpha\beta}(\varphi,\dot{\varphi})\ddot{\varphi}^{\beta}=\frac{\partial L}{\partial\varphi^{\alpha}}-\frac{\partial^2 L}{\partial\dot{\varphi}^{\alpha}\partial\varphi^{\beta}}\dot{\varphi}^{\beta}=a_{\alpha}(\varphi,\dot{\varphi}),\qquad \alpha=1,2,\cdots,n.\tag{3}$$

显然,当

$$|H_{\alpha\beta}(\varphi,\dot{\varphi})|\neq 0\tag{4}$$

时,从式(3)可以得到关于 $\ddot{\varphi}$ 的所有解.此时,系统 Lagrange 函数的 Hesse 矩阵为非退化的,相应的系统称为常规系统.

本文仅考虑系统 Hesse 矩阵退化的情况,即

$$|H_{\alpha\beta}(\varphi,\dot{\varphi})|=0, \tag{5}$$

当系统转化到相空间 Hamilton 描述时,广义动量坐标的定义为

$$\pi_\alpha=\frac{\partial L}{\partial\dot{\varphi}^\alpha},\qquad \alpha=1,2,\cdots,n. \tag{6}$$

当系统的 Hesse 矩阵非退化时,通过式(3)和(6)可以得到所有的 $\ddot{\varphi}^\alpha$,其中 $\ddot{\varphi}^\alpha$ 是作为 φ 和 π 的函数.反之,当 Hesse 矩阵退化的时候就不能解出所有的 $\ddot{\varphi}^\alpha$.因此就必须考虑系统的内在约束.系统的 Hesse 矩阵行列式为 0,设奇异系统 Hesse 矩阵的秩为 $R(R<n)$,则从式(3) 可以计算出系统的正则变量 φ 和 $\dot{\varphi}$ 之间存在着 $n-R$ 个关系式:

$$\xi_\alpha^i(\varphi,\dot{\varphi})=0,\qquad \alpha=1,2,\cdots,n-R. \tag{7}$$

我们称这些关系式为系统的 Lagrange 约束,即固有约束.为了避免这些约束会产生进一步的自洽性情况,把约束分为以下两类:

$$\begin{cases}A: A_\alpha(\varphi)=0,\\ B: B_\lambda(\varphi,\dot{\varphi})=0.\end{cases} \tag{8}$$

1.2 约束 Hamilton 系统的正则方程

假设 Hesse 矩阵的秩为 $R(R<n)$,则可以得到 R 个 $\dot{\varphi}^\alpha$ 作为函数 π_a,φ^α 和剩余的 $\dot{\varphi}^\rho$ 的函数:

$$\dot{\varphi}^\sigma=f^\sigma(\varphi,\pi_a,\dot{\varphi}^\rho), \tag{9}$$

其中 $\sigma=1,2,\cdots,n;\rho=R+1,\cdots,n;a=1,2,\cdots,R$.然后把式(5)代入式(7)中可以得到

$$\pi_i=g_i(\varphi,\pi_a,\dot{\varphi}^\rho). \tag{10}$$

当 $i=1,2,\cdots,R$ 时,方程(10) 是一个恒等式,当 $i=\rho$ 时,剩余的 $n-R$ 个 g_ρ 将不再依赖于 $\dot{\varphi}^\rho$.因此,可以得到 $n-R$ 个广义坐标和正则动量之间的关系式:

$$\phi_\alpha^0(\varphi,\pi)=\pi_\alpha-\varphi^\alpha(\varphi,\pi_\alpha),\qquad \alpha=1,2,\cdots,n-R. \tag{11}$$

系统的 Hamilton 量为

$$H=\pi_\alpha\dot{\varphi}^\alpha-L, \tag{12}$$

再引入 Lagrange 乘子 $\lambda^\alpha(t)$, 通过式(11)、(12)可以得到系统的 Hamilton 正则方程:

$$\dot{\varphi}^\alpha=\frac{\partial H}{\partial\pi_\alpha}+\lambda^\alpha\frac{\partial\phi_\beta^0}{\partial\pi_\alpha}, \tag{13}$$

$$\dot{\pi}_\alpha=-\frac{\partial H}{\partial\varphi^\alpha}-\lambda^\alpha\frac{\partial\phi_\beta^0}{\partial\varphi^\alpha}, \tag{14}$$

其中 $\phi_\alpha^0(\varphi,\pi)=0;\alpha=1,2,\cdots,n;\ \beta=1,2,\cdots,n-R$.称方程(13)、(14)为相应约束 Hamilton 系统的自由系统的正则运动方程.

考虑仅含有第二类约束的奇异系统,即假设约束(11)包含第二类初级约束和次级约束,有

$$\det\{\phi_i^0,\phi_j^0\}_{\phi=0}\neq 0,\qquad i\neq j;\ i,j=1,2,\cdots,n-r.$$

那么式(13)、(14)中所有 Lagrange 乘子 λ_j 可由约束的相容许条件

$$\{\phi_i^0,H_{\mathrm{T}}\}=\{\phi_i^0,H\}+\lambda_i\{\phi_i^0,\phi_j^0\}=0 \tag{15}$$

完全确定,其中

$$H_{\mathrm{T}}=H+\lambda^\alpha\phi_\alpha^0.$$

2 约束 Hamilton 系统的无限小变换和 Lie 对称性

引入关于空间位置和态函数的无限小变换(变换 Lie 群):

$$\begin{cases}x^* = x + \varepsilon\xi_0(x,\varphi^\alpha,\pi_\alpha), \\ \varphi^{\alpha *} = \varphi^\alpha + \varepsilon\xi_\alpha(x,\varphi^\alpha,\pi_\alpha), \\ \pi_\alpha^* = \pi + \varepsilon\eta_\alpha(x,\varphi^\alpha,\pi_\alpha),\end{cases} \tag{16}$$

其中 ε 为无限小参数, $\xi_0,\xi_\alpha,\eta_\alpha$ 为无限小变换的生成元, 下面构造无限小生成元计算符及其一次扩展和二次扩展 $X^{(0)},X^{(1)},X^{(2)}$.

$$X^{(0)} = \xi_0\frac{\partial}{\partial x} + \xi_\alpha\frac{\partial}{\partial\varphi^\alpha} + \eta_\alpha\frac{\partial}{\partial\pi_\alpha}, \tag{17}$$

它的一次扩展为

$$X^{(1)} = X^{(0)} + (\dot{\xi}_\alpha - \dot{\varphi}^\alpha\dot{\xi}_0)\frac{\partial}{\partial\dot{\varphi}^\alpha} + (\dot{\eta}_\alpha - \dot{\pi}_\alpha\dot{\xi}_0)\frac{\partial}{\partial\dot{\pi}_\alpha}. \tag{18}$$

根据微分方程在无限小变换下的不变性理论得知, 方程(13)和(14)在无限小变换下的不变性归为

$$\begin{cases}X^{(1)}(\dot{\varphi}^\alpha - g_\alpha(x,\varphi,\pi)) = 0, \\ X^{(1)}(\dot{\pi}_\alpha - h_\alpha(x,\varphi,\pi)) = 0.\end{cases} \tag{19}$$

如果

$$\dot{\varphi}^\alpha = g_\alpha,\ \dot{\pi}_\alpha = h_\alpha,$$

将算子(18)代入方程(19)并注意到方程(17), 可得

$$\begin{cases}\dot{\xi}_\alpha - \dot{\xi}_0 g_\alpha = X^{(0)}(g_s), \\ \dot{\eta}_\alpha - \dot{\xi}_0 h_\alpha = X^{(0)}(h_s),\end{cases} \quad \alpha = 1,2,\cdots,n, \tag{20}$$

即

$$\begin{cases}\dot{\xi}_\alpha - \dot{\xi}_0 g_\alpha = X^{(0)}\left(\dfrac{\partial H_c}{\partial\pi_\alpha}\right) + X^{(0)}\left(\lambda_j\dfrac{\partial\phi_j^0}{\partial\pi_\alpha}\right), \\ \dot{\eta}_\alpha - \dot{\xi}_0 h_\alpha = -X^{(0)}\left(\dfrac{\partial H_c}{\partial\varphi^\alpha}\right) - X^{(0)}\left(\lambda_j\dfrac{\partial\phi_j^0}{\partial\varphi^\alpha}\right),\end{cases} \quad \alpha = 1,2,\cdots,n. \tag{21}$$

约束(11)在无限小变换(16)下的不变性归结为

$$X^{(0)}(\phi_j^0(\varphi^\alpha,\pi_\alpha))\big|_{\phi_\alpha^0=0} = 0, \qquad j = 1,2,\cdots,n-r. \tag{22}$$

式(21)称为无限小生成元的确定方程, 式(22)称为无限小生成元的限制方程.

定义 1 如果无限小生成元 ξ_0, ξ_α 和 η_α 满足确定方程(21), 则相应的对称性为与约束 Hamilton 系统相应的自由 Hamilton 系统的 Lie 对称性.

定义 2 如果无限小生成元 ξ_0, ξ_α 和 η_α 满足确定方程(21)和限制方程(22), 则相应的对称性为约束 Hamilton 系统(13)、(14)的弱 Lie 对称性.

单从微分方程在无限小变换下的不变性考虑, 上述定义的 Lie 对称性就是通常理解的 Lie 对称性. 但是, 若考虑微分方程的导出过程, 则对无限小生成元还要施加另外的限制, 而必须定义另外的 Lie 对称性. 在推导式(22)时, 有

$$\frac{\partial\phi_j^0}{\partial\pi_\alpha}\delta\pi_\alpha + \frac{\partial\phi_j^0}{\partial\varphi^\alpha}\delta\varphi^\alpha = 0. \tag{23}$$

将由变换(16)确定的变分代入式(23)得

$$\frac{\partial\phi_j^0}{\partial\varphi^\alpha}(\xi_\alpha - \dot{\varphi}^\alpha\xi_0) + \frac{\partial\phi_j^0}{\partial\pi_\alpha}(\eta_\alpha - \dot{\pi}_\alpha\xi_0) = 0. \tag{24}$$

称式(24)为附加限制方程.

定义 3　如果无限小生成元 ξ_0, ξ_α 和 η_α 满足确定方程(21)、限制方程(22)和附加限制方程(24),则称相应的对称性为约束 Hamilton 系统(13)、(14)的强 Lie 对称性.

3　系统中的结构方程和守恒定理

对于奇异系统,Lie 对称性不一定会导致守恒量.下面给出 Lie 对称性导致守恒量的条件以及守恒量的形式.

定理 1　对于满足确定方程(21)的无限小生成元 ξ_0, ξ_α 和 η_α,如果存在规范函数 $G = G(x,\varphi,\pi)$ 满足如下结构方程:

$$-H_c\dot{\xi}_0 + \dot{\varphi}^\alpha + \pi_\alpha\dot{\xi}_\alpha - X^{(0)}(H + c) - \lambda_j\frac{\partial\phi_j^0}{\partial\varphi^\alpha}(\xi_\alpha - \dot{\varphi}^\alpha\xi_0) - \lambda_j\frac{\partial\phi_j^0}{\partial\pi_\alpha}(\eta_\alpha - \dot{\pi}_\alpha\xi_0) + \dot{G} = 0, \tag{25}$$

则与约束 Hamilton 系统相应的自由 Hamilton 系统(13)、(14)存在如下形式的 Lie 对称性守恒量:

$$I = - H_c\xi_0 + \pi_\alpha\xi_\alpha + G = \mathrm{const}. \tag{26}$$

定理 2　对于满足确定方程(21)和限制方程(22)的无限小生成元 ξ_0, ξ_α 和 η_α,如果存在规范函数 $G = G(x,\varphi,\pi)$ 满足结构方程(26),则约束 Hamilton 系统(13)、(14)存在形如式(26)的弱 Lie 对称性守恒量.

定理 3　对于满足确定方程(21)、限制方程(22)和附加限制方程(24)的无限小生成元 ξ_0,ξ_α 和 η_α,如果存在规范函数 $G = G(x,\varphi,\pi)$ 满足如下结构方程:

$$-H_c\dot{\xi}_0 + \dot{\varphi}^\alpha + \pi_\alpha\dot{\xi}_\alpha - X^{(0)}(H + c) + \dot{G} = 0, \tag{27}$$

则约束 Hamilton 系统(13)、(14)存在形如式(26)的强 Lie 对称性守恒量,以上定理均已得到了证明.

4　算　　例

4.1　例 1

复标量场耦合 Chern-Simons 项在(1+2)维时空中的 Lagrange 密度为

$$\tilde{L} = (\mathrm{D}_\mu\varphi)^*(\mathrm{D}^\mu\varphi) + \frac{k}{4\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda,$$

其中

$$\mathrm{D}_\mu = \partial_\mu - \mathrm{i}A_\mu,\ \varepsilon_{012} = 1,$$

容易看出,式中的 Lagrange 量是奇异的.

场量 A_μ,φ,φ^* 的正则共轭动量分别为

$$\begin{cases}\pi_0 = \dfrac{\partial\tilde{L}}{\partial\dot{A}_0} = 0,\ \pi_\alpha = \dfrac{\partial\tilde{L}}{\partial\dot{A}_\alpha} = \dfrac{k}{4\pi}\varepsilon^{\alpha\beta}A_\beta, \\ \pi = \dfrac{\partial\tilde{L}}{\partial\dot{\varphi}} = (\mathrm{D}_0\varphi)^*,\ \pi^* = \dfrac{\partial\tilde{L}}{\partial\dot{\varphi}^*} = \mathrm{D}_0\varphi,\end{cases} \tag{28}$$

其中

$$J = \mathrm{i}(\pi\varphi - \varphi^*\pi^*).$$

此系统含有两个约束:

$$\begin{cases}\phi^1 = \pi_0 = 0,\\ \phi^2 = \dfrac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta - J_0.\end{cases} \tag{29}$$

系统的正则 Hamilton 量为

$$\tilde{H}_c = \pi^\mu \dot{A}_\mu + \pi\dot{\varphi} + \pi^*\varphi^* - \tilde{L} = \\ \pi\pi^* - (\mathrm{D}_\alpha)^*(\mathrm{D}^\alpha\varphi) - A_0\left(\frac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta - J_0\right). \tag{30}$$

总 Hamilton 函数为

$$H_{\mathrm{T}} = \int_V (\pi^\mu \dot{A}_\mu + \pi\dot{\varphi} + \pi^*\varphi^* - \tilde{L})\mathrm{d}^3x = \\ \int_V \left[\pi\pi^* - (\mathrm{D}_\alpha)^*(\mathrm{D}^\alpha\varphi) - A_0\left(\frac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta - J_0\right) + \lambda_1\phi^1 + \lambda_2\varphi^2\right]\mathrm{d}^3x. \tag{31}$$

由约束的相容许条件(15)得

$$\lambda_1 = A_0 + \varphi,\ \lambda_2 = A_0 + \varphi^*. \tag{32}$$

正则方程为

$$\begin{cases}\dot{A}_0 = \dfrac{\partial\tilde{H}}{\partial\pi_0} + \lambda_j\dfrac{\partial\phi_j}{\partial\pi_0} = \lambda_1,\ \dot{A}_\alpha = \dfrac{\partial\tilde{H}}{\partial\pi_\alpha} + \lambda_j\dfrac{\partial\phi_j}{\partial\pi_\alpha} = 0,\\ \dot{\varphi} = \dfrac{\partial\tilde{H}}{\partial\pi} + \lambda_j\dfrac{\partial\phi_j}{\partial\pi} = \pi^* - \mathrm{i}A_0\varphi - \mathrm{i}\lambda_2\varphi,\\ \dot{\varphi}^* = \dfrac{\partial\tilde{H}}{\partial\pi^*} + \lambda_j\dfrac{\partial\phi_j}{\partial\pi^*} = \pi - \mathrm{i}A_0\varphi^* + \lambda_1 + \mathrm{i}\lambda_2\varphi^*,\end{cases} \tag{33}$$

$$\begin{cases}\dot{\pi}_0 = -\dfrac{\partial\tilde{H}}{\partial A_0} - \lambda_j\dfrac{\partial\phi_j}{\partial A_0} = \left(\dfrac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta - J_0\right) - \lambda_1,\\ \dot{\pi}_\alpha = -\dfrac{\partial\tilde{H}}{\partial A_\alpha} - \lambda_j\dfrac{\partial\phi_j}{\partial A_\alpha} = 0,\\ \dot{\pi} = -\dfrac{\partial\tilde{H}}{\partial\varphi} - \lambda_j\dfrac{\partial\phi_j}{\partial\varphi} = \mathrm{i}A_0\pi + \mathrm{i}\lambda_2\pi,\\ \dot{\pi}^* = -\dfrac{\partial\tilde{H}}{\partial\varphi^*} - \lambda_j\dfrac{\partial\phi_j}{\partial\varphi^*} = \mathrm{i}A_0\pi^* - \mathrm{i}\lambda_2\pi^*.\end{cases} \tag{34}$$

确定方程给出

$$\begin{cases}\dot{\xi}_1 - (A_0 + \varphi^* - \pi)\dot{\xi}_0 = \\ \quad \xi_0(A_0 + \varphi^* - 2\pi + \pi^* - 2iA_0\varphi - \mathrm{i}\pi^*\varphi) + \xi_1\cdot 1 + \eta_1\cdot 0,\\ \dot{\xi}_2 - 0\cdot\xi_0 = 0,\\ \dot{\xi}_3 - (\pi^* - 2\mathrm{i}A_0\varphi - \mathrm{i}\pi^*\varphi)\dot{\xi}_0 = \\ \quad \xi_0(-2\mathrm{i}\pi^*\pi^* - 2\mathrm{i}A_0\varphi - 2\mathrm{i}\varphi^*\varphi + 2\mathrm{i}\pi\varphi - 2\mathrm{i}A_0\pi^* + 4\mathrm{i}^2{A_0}^2\varphi) + \\ \quad 4\mathrm{i}^2A_0\pi^*\varphi + 2\mathrm{i}^*\pi^{*2}\varphi + \xi_3(-2\mathrm{i}A_0 - \mathrm{i}\pi^*) + \eta_3\cdot 0,\\ \dot{\xi}_4 - (A_0 + \varphi^* + \mathrm{i}\pi^*\varphi^*)\dot{\xi}_0 = \\ \quad \xi_0(2A_0 + 2\varphi^* - \pi + 2\mathrm{i}\pi^*\varphi^* + \mathrm{i}\pi^*A_0) + \xi_4(\mathrm{i}\varphi^*) + \eta_4(1 + \mathrm{i}\pi^*),\end{cases} \tag{35}$$

$$\begin{cases}\dot{\eta}_1-\left(\dfrac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta-\mathrm{i}\pi\varphi+\mathrm{i}\varphi^*\pi^*-A_0-\varphi^*+\pi\right)\dot{\xi}_0=\\ \qquad -\xi_0(\mathrm{i}\pi^*A_0-2\mathrm{i}A_0+\pi+2\mathrm{i}A_0\pi)-\xi_1\cdot(-1)+\eta_1\cdot 0,\\ \dot{\eta}_2-0\cdot\dot{\xi}_0=0,\\ \dot{\eta}_3-(2\mathrm{i}A_0\pi+\mathrm{i}\pi^*\pi)\dot{\xi}_0=\\ \qquad -\xi_0(2\mathrm{i}\pi A_0+2\mathrm{i}\pi\varphi^*-2\mathrm{i}\pi\pi+4\mathrm{i}^2A_0{}^2\pi)-\xi_3\cdot 0-\eta_3(2\mathrm{i}A_0+\mathrm{i}\pi^*),\\ \dot{\eta}_4-(-\mathrm{i}\pi^*\pi^*)\dot{\xi}_0=-\xi_0(2\mathrm{i}^2\pi^{*3}-\xi_4\cdot 0-\eta_4(-2\mathrm{i}\pi^*).\end{cases} \tag{36}$$

其解如下:

$$\begin{cases}\xi_0=0,\ \xi_1=\mathrm{e}^x,\ \xi_2=\mathrm{const},\ \xi_3=\mathrm{e}^{(2\mathrm{i}A_0-2\mathrm{i}\pi^*)x},\ \xi_4=(C_1+C_2x)\mathrm{e}^{\mathrm{i}\varphi^*x},\\ \eta_1=\mathrm{e}^x,\ \eta_2=\mathrm{const},\ \eta_3=\mathrm{e}^{(2\mathrm{i}\pi^*-2\mathrm{i}A_0)x},\ \eta_4=\mathrm{e}^{2\mathrm{i}\pi^*x}.\end{cases} \tag{37}$$

限制方程给出

$$X^0(\pi_0)=0,\ X^0\left(\frac{k}{2\pi}\varepsilon^{\alpha\beta}\partial_\alpha A_\beta-J_0\right)=0. \tag{38}$$

附加限制方程给出

$$\begin{cases}\dfrac{\partial\pi_0}{\partial A_0}(\xi_1-\dot{A}_0\xi_0)+\dfrac{\partial\pi_0}{\partial\pi_0}(\eta_1-\dot{\pi}_0\xi_0)=0,\\ \dfrac{\partial\pi_0}{\partial A_\alpha}(\xi_2-\dot{A}_\alpha\xi_0)+\dfrac{\partial\pi_0}{\partial\pi_\alpha}(\eta_2-\dot{\pi}_\alpha\xi_0)=0,\\ \dfrac{\partial\pi_0}{\partial\varphi}(\xi_3-\dot{\varphi}\xi_0)+\dfrac{\partial\pi_0}{\partial\varphi}(\eta_3-\dot{\pi}\xi_0)=0,\\ \dfrac{\partial\pi_0}{\partial\varphi^*}(\xi_4-\dot{\varphi}^*\xi_0)+\dfrac{\partial\pi_0}{\partial\varphi^*}(\eta_4-\dot{\pi}^*\xi_0)=0,\end{cases} \tag{39}$$

$$\begin{cases}\dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial A_0}(\xi_1-\dot{A}_0\xi_0)+\\ \qquad\dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\pi_0}(\eta_1-\dot{\pi}_0\xi_0)=0,\\ \dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial A_\alpha}(\xi_2-\dot{A}_\alpha\xi_0)+\\ \qquad\dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\pi_\alpha}(\eta_2-\dot{\pi}_\alpha\xi_0)=0,\\ \dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\varphi}(\xi_3-\dot{\varphi}\xi_0)+\\ \qquad\dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\varphi}(\eta_3-\dot{\pi}\xi_0)=0,\\ \dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\varphi^*}(\xi_4-\dot{\varphi}^*\xi_0)+\\ \qquad\dfrac{\partial((k/(2\pi))\varepsilon^{\alpha\beta}\partial_\alpha A_\beta+\mathrm{i}\varphi^*\pi^*-\mathrm{i}\pi\varphi)}{\partial\varphi^*}(\eta_4-\dot{\pi}^*\xi_0)=0.\end{cases} \tag{40}$$

把生成函数(34)代入结构方程(25),可以得到规范函数:

$$G = A_0 e^x + \varphi e^{(2iA_0 - 2i\pi)x}, \tag{41}$$

相应系统的守恒量为

$$I = \pi_0 e^x + \pi e^{(2i\pi^* - 2iA_0 x)} + \pi^* e^{2i\pi^* x} + A_0 e^x + \varphi e^{(2iA_0 - 2i\pi)x} = \text{const}. \tag{42}$$

很容易验证生成元(37)不满足条件(38)、(39),所以生成元是相应自由系统的一般 Lie 变换生成元,则守恒量是相应自由系统的一般 Lie 对称性守恒量.

4.2　例 2

奇异系统中 Lagrange 量:

$$L(\varphi^1, \varphi^2, \dot{\varphi}^1, \dot{\varphi}^2) = \frac{1}{2}(\dot{\varphi}^1)^2 + \dot{\varphi}^1\varphi^2 + \frac{1}{2}(\varphi^1 - \varphi^2)^2,$$

试求其系统的 Lie 对称性及守恒量.

系统的广义动量为

$$\begin{cases} \pi_1 = \dfrac{\partial L}{\partial \dot{\varphi}^1} = \dot{\varphi}^1 + \varphi^2, \\ \pi_2 = \dfrac{\partial L}{\partial \dot{\varphi}^2} = 0. \end{cases}$$

Lagrange 函数 L 的 Hesse 矩阵的秩 $r = 1$, 从而正则变量间存在一个固有约束:

$$\phi = \pi_1 - \dot{\varphi}^1 - \varphi^2 = 0. \tag{43}$$

系统的正则 Hamilton 函数为

$$\tilde{H}_c = \dot{\varphi}^1\pi_1 + \dot{\varphi}^2\pi_2 - \tilde{L} = \frac{1}{2}\pi_1{}^2 + \varphi^1\varphi^2 - \frac{1}{2}(\varphi^1)^2 - \pi_1\varphi^2. \tag{44}$$

总 Hamilton 函数为

$$H_T = \dot{\varphi}^1\pi_1 + \dot{\varphi}^2\pi_2 - \tilde{L} = \frac{1}{2}\pi_1{}^2 + \varphi^1\varphi^2 - \frac{1}{2}(\varphi^1)^2 - \pi_1\varphi^2 + \lambda(\pi_1 - \varphi^1 - \varphi^2). \tag{45}$$

由约束的相容许条件得到

$$\lambda = \dot{\varphi}^1 - \varphi^1 + \varphi^2. \tag{46}$$

系统的运动微分方程给出

$$\begin{cases} \dot{\varphi}^1 = \pi_1 - \varphi^2,\ \dot{\varphi}^2 = 0, \\ \dot{\pi}_1 = \varphi^1 - \varphi^2,\ \dot{\pi}_2 = 0. \end{cases} \tag{47}$$

确定方程(21)给出

$$\begin{cases} \dot{\xi}_1 - \dot{\varphi}^1\dot{\xi}_0 = \xi_0(\dot{\pi}_1 - \dot{\varphi}^2) + \eta_1 + \xi_0(\dot{\pi}_1 - \dot{\varphi}^1) - \xi_1, \\ \dot{\xi}_2 - \dot{\varphi}^2\dot{\xi}_0 = 0, \\ \dot{\eta}_1 - \dot{\pi}_1\dot{\xi}_0 = -\xi_0(\dot{\varphi}^2 - \dot{\varphi}^1) + \xi_1, \\ \dot{\eta}_2 - \dot{\pi}_2\dot{\xi}_0 = -\xi_0(\dot{\varphi}^1 - \dot{\pi}_1) + \xi_0(\dot{\pi}_1 - \dot{\varphi}^1) - \xi_2. \end{cases} \tag{48}$$

则有如下解:

$$\xi_0 = 0,\ \xi_1 = \sin x - \cos x,\ \xi_2 = c,\ \eta_1 = \cos x,\ \eta_2 = -cx. \tag{49}$$

限制方程(22)给出

$$\begin{cases}\xi_0(\dot{\pi}_1-\dot{\varphi}^1+\dot{\varphi}^2)+\eta_1=0,\\ \xi_0(\dot{\pi}_1-\dot{\varphi}^1+\dot{\varphi}^2)-\xi_2=0.\end{cases}\tag{50}$$

附加限制方程(24)给出

$$\begin{cases}\eta_1-\dot{\pi}_1\xi_0=0,\\ \dot{\varphi}^2\xi_0-\xi_1=0.\end{cases}\tag{51}$$

生成元对应的规范函数为

$$\eta_1(\pi_1-\varphi^1)+\dot{\xi}_1\pi_1+\dot{\xi}_2\pi_2-\xi_1(\varphi^2-\varphi^1)-\xi_2(\varphi^1-\pi_1)-\eta_1(\pi_1-\varphi^2)+\xi_2(\pi_1-\varphi^1)-\eta_1(\pi_1-\varphi^1)+\dot{G}=0.\tag{52}$$

解得

$$G=\pi_1\cos x+\varphi^1\sin x+2(\varphi^1-\pi_1)^2.\tag{53}$$

对应的守恒量为

$$I=\pi_1\sin x+\varphi^1\sin x+c\pi_2+2(\varphi^1-\pi_1)^2.\tag{54}$$

容易验证,生成元(49)不满足条件(50)、(51),因此它是相应的自由 Hamilton 系统的 Lie 对称变换生成元,则守恒量为相应自由 Hamilton 系统的 Lie 对称性守恒量.

5 结 论

本文采用 Lie 对称性方法,给出了约束 Hamilton 系统的守恒量定理,并首次把这种方法延伸到场论中,从而扩展了 Lie 对称性方法的应用空间,使得对称性理论体系更加完善.给出了满足 Lie 确定方程的无穷小变换生成元,分别定义了弱 Lie 对称性和强 Lie 对称性,其根据是无穷小生成元是否满足约束的限制方程和附加限制方程,从而相应的守恒量又可以分为弱 Lie 对称性守恒量和强 Lie 对称性守恒量.而且依据 Lie 对称性的思想研究了场论的守恒定理,探究了复标量场耦合 Chern-Simons 项系统的守恒定理,从而得到其守恒量.

Lie 对称性方法还可以应用到规范场和非规范场、Abel 场和非 Abel 场、杨-Mills 等量子场论中.笔者接下来的工作将会是应用 Lie 对称性方法并结合冯康先生的辛算法数值模拟工具,对这些系统做进一步的研究.

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Lie Symmetry of Constrained Hamiltonian Systems and Its Application in Field Theory

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Abstract: The Lie symmetry method was studied for constrained Hamiltonian systems, and the conservation laws of the field theory systems were obtained. Firstly, the generalized canonical equations for constrained Hamiltonian systems were derived. Secondly, the determining equations and structural equations about the Lie symmetry of the constrained Hamiltonian systems were deduced. Thirdly, the Lie theorems and the conserved quantities for constrained Hamiltonian systems were given. Finally, the Lie symmetry for the system of the complex scalar field coupled to the Chern-Simons term was discussed. Two examples in the field theory illustrate the validity of this method.

Key words: Lie symmetry; constrained Hamiltonian system; field theory; conserved quantity

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