

# 箱形梁剪力滞效应的分离求解方法 及参数影响分析\*

张玉元, 张元海, 张慧

(兰州交通大学 土木工程学院, 兰州 730070)

**摘要:** 选取剪力滞效应引起的附加挠度为广义位移,将箱梁的剪力滞变形状态从初等梁挠曲变形状态中分离出来,作为一种独立的变形状态进行分析,运用能量变分法建立了以附加挠度为未知量的截面控制微分方程及边界条件;结合简支边界条件分别导出了集中荷载和均布荷载作用下箱梁的附加挠度和纵向应力计算公式.纵向应力分析表明:该文方法计算的应力结果和样条函数法计算的应力结果吻合良好,从而验证了其方法的正确性.挠度研究表明:剪力滞附加挠度由跨中向两侧支点递减;针对于该文算例而言,均布荷载和集中荷载作用下跨中截面的剪力滞附加挠度分别占初等梁挠度的2.57%和3.03%;随着高跨比和宽跨比的增大,相应箱梁跨中截面的附加挠度逐渐减小,且宽跨比对附加挠度的影响远大于高跨比的影响.

**关键词:** 薄壁箱梁; 剪力滞翘曲; 初等梁理论; 附加挠度; 能量变分法

**中图分类号:** U448.213

**文献标志码:** A

**DOI:** 10.21656/1000-0887.380073

## 引 言

薄壁箱梁因其有利的受力特性而被广泛应用于现代桥梁工程中.薄壁箱梁发生竖向挠曲变形时,由腹板传递给翼缘板的剪力流使翼缘板在远离腹板处的纵向位移滞后于靠近腹板处的纵向位移,从而使箱梁翼缘板不满足平截面假设,这就是剪力滞效应<sup>[1-2]</sup>.在薄壁箱梁的设计和计算时考虑剪力滞效应这已是众所周知<sup>[1,3-5]</sup>.

Reissner<sup>[6]</sup>针对不带悬臂板的双轴对称矩形截面箱梁,选取最大剪切转角差为广义位移,首次应用能量变分法建立了剪力滞效应分析理论.钱寅泉等<sup>[7-8]</sup>在Reissner的基础上,将矩形箱梁进一步拓展到带有悬臂板的斜腹板箱梁中,选取余弦函数为翘曲位移函数,利用能量变分法分析箱梁剪力滞效应.近年来,对翘曲正应力自平衡条件主要有以下两种途径:途径一,翘曲正应力在箱梁横截面上合成的轴力为零,此途径国内外学者采用居多;途径二,翘曲正应力在竖向弯曲面内合成的弯矩为零,此途径应用较少.文献[4,7-12]中采用途径一,在箱梁翘曲位移中加入均匀附加位移,使其进一步满足轴向自平衡条件;文献[13-14]中采用途径二,对槽形宽梁底板和腹板的纵向位移函数进行修正.关于剪力滞广义位移的选取有以下两种方式:最大

\* 收稿日期: 2017-03-29; 修订日期: 2017-12-05

**基金项目:** 国家自然科学基金(51468032;51268029;51508255);2017年度陇原青年创新创业人才资助项目

**作者简介:** 张玉元(1989—),男,博士生(E-mail: zyy161208@163.com);

张元海(1965—),男,教授,博士,博士生导师(通讯作者. E-mail: zyh17012@163.com).

剪切转角差和附加挠度,文献[5-8,10-14]中箱梁各板采用同一最大剪切转角差为广义位移研究剪力滞效应,此方式比较传统,应用也较为广泛;文献[15]中考虑到箱梁挠曲变形时各板的纵向翘曲位移存在差异,因此各板选取不同的最大剪切转角差为广义位移,这对于剪力滞效应精度的提高有一定的影响,但计算较为麻烦,且不具有明确的物理意义,不便于工程人员使用.文献[1,16]中采用剪力滞效应引起的附加挠度为广义位移,该广义位移具有明确的物理意义,且便于工程人员的理解和应用.此外,现有诸多文献在求解剪力滞效应时,将初等梁理论和剪力滞翘曲理论耦合在一起分析,这样不仅不能明确反映剪力滞效应对初等梁理论的影响程度,而且不便于设计人员应用.

虽然国内外学者对剪力滞效应的研究已取得了比较丰富的成果,但对剪力滞效应和初等梁理论解耦分析的相关文献较少.因此,本文将箱形梁的挠曲变形分解成剪力滞翘曲变形和初等梁弯曲变形,选取剪力滞效应引起的附加挠度为广义位移,应用能量变分法建立剪力滞效应分析理论.应用本文导出的计算公式分析简支箱梁在集中荷载和均布荷载下附加挠度和总挠度的分布曲线,以及高跨比、宽跨比对附加挠度的影响规律.

### 1 微分方程及边界条件

如图1所示,坐标系为  $Oxyz$ , 箱梁在竖向对称荷载作用下的挠曲变形位移模式(图1(a)),可分解为初等梁弯曲变形(图1(b))和剪力滞翘曲变形(图1(c)).针对剪力滞翘曲位移模式(图1(c)),假设坐标中心位于截面形心处,腹板服从平截面假定.

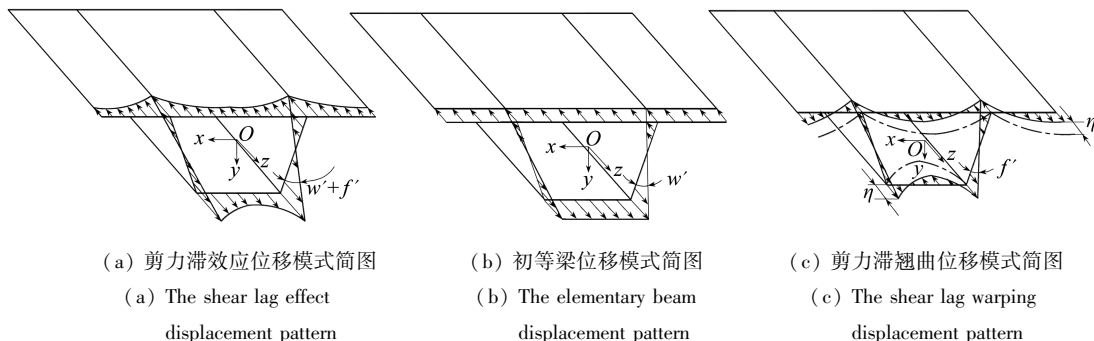


图1 箱梁截面的纵向位移组成

Fig. 1 The constitution of the longitudinal displacement of a box girder

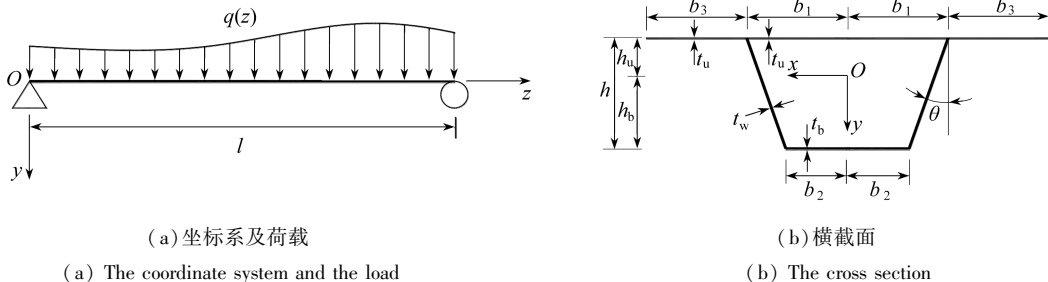


图2 箱型截面简图

Fig. 2 A box girder cross section

如图2所示,箱梁在任意竖向荷载  $q(z)$  作用下发生挠曲变形,选取剪力滞效应引起的附

加挠度为广义位移,则箱梁截面任一点处的纵向位移  $u(x, y, z)$  为

$$u(x, y, z) = u_0(y, z) + u_\omega(x, y, z) = -y \cdot w'(z) - \omega_\zeta(x, y) \cdot f'(z), \quad (1)$$

式中,  $u_0(y, z)$  为初等梁纵向位移,  $u_\omega(x, y, z)$  为剪力滞翘曲位移,  $w(z)$  为初等梁挠度,  $\omega_\zeta(x, y)$  为剪力滞翘曲位移函数,  $f(z)$  为剪力滞附加挠度。

图 2 中,  $b_1$  为顶板半宽,  $b_2$  为底板半宽,  $b_3$  为悬臂板宽,  $h_u$  为形心至顶板中面的距离,  $h_b$  为形心至底板中面的距离,  $h$  为顶底板中面之间的距离,  $t_u$  为顶板、悬臂板厚度,  $t_b$  为底板厚度,  $t_w$  为腹板厚度,  $\theta$  为腹板俯角, 计算悬臂板时将  $x$  轴坐标原点放至翼缘板右侧端部。

在  $Oxyz$  坐标系中, 各板的剪力滞翘曲位移函数  $\omega_\zeta(x, y)$  可表示为

$$\omega_\zeta(x, y) = \begin{cases} h_u \left( \cos \frac{\pi x}{2b_1} - \eta \right), & \text{top plate,} \\ -h_b \left( \cos \frac{\pi x}{2b_2} - \eta \right), & \text{bottom plate,} \\ h_u \left( \cos \frac{\pi x}{2b_3} - \eta \right), & \text{flange plate,} \\ \eta y, & \text{web plate,} \end{cases} \quad (2)$$

式中,  $\eta$  为翘曲应力自平衡修正系数, 将通过翘曲应力在面内合成的弯矩为零来确定。

根据几何方程及 Hooker(胡克)定律, 由式(1)可得箱梁截面任一点处的纵向应力  $\sigma(x, y, z)$  为

$$\sigma(x, y, z) = \sigma_0(y, z) + \sigma_\omega(x, y, z) = -E(y \cdot w''(z) + \omega_\zeta(x, y) \cdot f''(z)), \quad (3)$$

式中,  $\sigma_0(y, z)$  为初等梁应力, 将通过材料力学弯曲正应力计算公式求得;  $\sigma_\omega(x, y, z)$  为剪力滞翘曲应力;  $E$  为弹性模量。

箱梁在竖向挠曲变形时, 截面正应力合成的弯矩为零, 即

$$\int_A \sigma \cdot y dA = 0. \quad (4)$$

将式(3)代入式(4)可得

$$\int_A \omega_\zeta \cdot y dA = 0. \quad (5)$$

经积分运算可得翘曲应力自平衡修正系数  $\eta$ :

$$\eta = \frac{2(I_1 + I_2 + I_3)}{\pi \cdot I}, \quad (6)$$

式中,  $I$  为箱梁截面惯性矩,  $I_i$  为相应各板的截面惯性矩, 下标  $i$  取 1, 2, 3,  $w$  分别代表顶板、底板、悬臂板和腹板(下同), 计算公式为  $I_1 = 2t_u b_1 h_u^2$ ,  $I_2 = 2t_b b_2 h_b^2$ ,  $I_3 = 2t_u b_3 h_u^2$ ,  $I_w = 2t_w (h_u^3 + h_b^3) / (3 \cos \theta)$ ,  $I = I_1 + I_2 + I_3 + I_w$ 。

由弹性力学可知, 箱梁各板的应变能可表达为

$$U_i = \frac{1}{2} \iiint (E \varepsilon_{iz}^2 + G \gamma_{ixz}^2) dx dy dz, \quad (7)$$

式中,  $\varepsilon_{iz} = \frac{\partial u_i(x, y, z)}{\partial z}$ ,  $\gamma_{ixz} \approx \frac{\partial u_i(x, y, z)}{\partial x}$ ,  $G$  为剪切模量。

则箱梁顶板、底板和悬臂板的应变能表达式为

$$U_i = \frac{EI_i}{2} \int_0^l \left[ w''^2 + \left( \frac{1}{2} + \eta^2 - \frac{4\eta}{\pi} \right) f''^2 - 2 \left( \frac{2}{\pi} - \eta \right) w'' f'' \right] dz + \frac{G\pi^2 I_i}{16b_i^2} \int_0^l f'^2 dz,$$

$$i = 1, 2, 3. \quad (8)$$

腹板应变能表达式为

$$U_w = \frac{EI_w}{2} \int_0^l (w''^2 + \eta^2 f''^2 + 2\eta w'' f'') dz. \quad (9)$$

外力势能为

$$V = - \int_0^l q(w + f) dz. \quad (10)$$

则箱梁的总势能可表达为

$$\begin{aligned} \Pi = U_1 + U_2 + U_3 + U_w + V = \\ E \int_0^l \left( \frac{I}{2} w''^2 + \zeta_1 f''^2 + \zeta_2 w'' f'' \right) dz + \frac{G\pi^2 \zeta_3}{16} \int_0^l f'^2 dz - \int_0^l q(w + f) dz, \end{aligned} \quad (11)$$

式中

$$\zeta_1 = \frac{I_s}{2} \left( \frac{1}{2} - \frac{4\eta}{\pi} \right) + \eta^2 \cdot \frac{I}{2}, \quad \zeta_2 = \eta I - \frac{2}{\pi} I_s, \quad \zeta_3 = \frac{I_1}{b_1^2} + \frac{I_2}{b_2^2} + \frac{I_3}{b_3^2}, \quad I_s = I_1 + I_2 + I_3.$$

总势能泛函求一阶变分,并令  $\delta\Pi = 0$ , 即

$$\begin{aligned} \delta\Pi = \int_0^l (EIw'''' + E\zeta_2 f'''' - q) \delta w dz + \int_0^l \left( 2E\zeta_1 f'''' + E\zeta_2 w'''' - \frac{G\pi^2 \zeta_3}{8} f'' - q \right) \delta f dz + \\ (EIw'' + E\zeta_2 f'') \delta w' \Big|_0^l + (2E\zeta_1 f'' + E\zeta_2 w'') \delta f' \Big|_0^l - \\ (EIw''' + E\zeta_2 f''') \delta w \Big|_0^l - \left( 2E\zeta_1 f''' + E\zeta_2 w''' - \frac{G\pi^2 \zeta_3}{8} f' \right) \delta f \Big|_0^l = 0. \end{aligned} \quad (12)$$

根据变分引理,可得箱梁截面控制微分方程如下:

$$EIw'''' + E\zeta_2 f'''' - q = 0, \quad (13)$$

$$2E\zeta_1 f'''' + E\zeta_2 w'''' - \frac{G\pi^2 \zeta_3}{8} f'' - q = 0. \quad (14)$$

联立式(13)、(14)消去  $w$ , 可得附加挠度的微分方程如下:

$$f'''' - k^2 f'' = k_1 q, \quad (15)$$

式中

$$k^2 = \frac{G\pi^2 \zeta_3}{8E(2\zeta_1 I - \zeta_2^2)}, \quad k_1 = \frac{I - \zeta_2}{E(2\zeta_1 I - \zeta_2^2)}.$$

关于微分方程式(15)通解的一般形式为

$$f = C_1 + C_2 z + C_3 \sinh(kz) + C_4 \cosh(kz) + f^*. \quad (16)$$

式(16)中的待定系数由边界条件求解,  $f^*$  为仅与  $q(z)$  分布有关的特解.

确定上述4个常数的边界条件为:

$$\text{固定端 } f = 0, f' = 0;$$

$$\text{简支端 } f = 0, f'' = 0;$$

$$\text{自由端 } f'' = 0, f''' - k^2 f' = 0.$$

## 2 纵向应力及挠度的求解

如图3所示,简支箱梁在均布荷载  $q$  作用下,箱梁的剪力滞附加挠度可表达为

$$f = C_1 + C_2 z + C_3 \sinh(kz) + C_4 \cosh(kz) - \frac{k_1 q}{2k^2} z^2. \quad (17)$$

为了确定上式中的 4 个参数,需利用以下 4 个边界条件:

$$f(0) = 0, f''(0) = 0, f(l) = 0, f''(l) = 0.$$

利用以上 4 个边界条件确定常数  $C_1 \sim C_4$  后,即可得到均布荷载作用下简支箱梁剪力滞附加挠度的计算公式:

$$f = \frac{k_1 q}{k^4} \left( \frac{1 - \cosh(kl)}{\sinh(kl)} \sinh(kz) + \cosh(kz) - 1 \right) + \frac{k_1 q}{2k^2} (lz - z^2). \quad (18)$$

由材料力学可知简支箱梁均布荷载作用下的挠度计算公式,将初等梁挠度和剪力滞附加挠度计算公式代入式(3)可得箱梁截面任一点处的纵向应力表达式,即

$$\sigma(x, y, z) = \frac{qyz}{2I} (l - z) - \frac{Ek_1 q \omega_\xi(x, y)}{k^2} \left( \cosh(kz) - 1 + \frac{1 - \cosh(kl)}{\sinh(kl)} \sinh(kz) \right). \quad (19)$$

如图 4 所示,简支箱梁承受跨中集中荷载  $P$ ,将集中荷载  $P$  按照文献[17]介绍的方法换算成均布荷载并取其前 4 项:

$$q(z) = \frac{2P}{l} \left( \sin \frac{\pi z}{l} - \sin \frac{3\pi z}{l} + \sin \frac{5\pi z}{l} - \sin \frac{7\pi z}{l} \right). \quad (20)$$

则箱梁的剪力滞附加挠度可表达为

$$f = C_1 + C_2 z + C_3 \sinh(kz) + C_4 \cosh(kz) - \frac{2Pk_1 l^3}{\pi^2} \left( \alpha_1 \sin \frac{\pi z}{l} - \alpha_2 \sin \frac{3\pi z}{l} + \alpha_3 \sin \frac{5\pi z}{l} - \alpha_4 \sin \frac{7\pi z}{l} \right), \quad (21)$$

式中

$$\alpha_1 = \frac{1}{\pi^2 + k^2 l^2}, \alpha_2 = \frac{1}{9(9\pi^2 + k^2 l^2)}, \alpha_3 = \frac{1}{25(25\pi^2 + k^2 l^2)}, \alpha_4 = \frac{1}{49(49\pi^2 + k^2 l^2)}.$$

为了确定上式中的 4 个参数,需利用以下 4 个边界条件:

$$f(0) = 0, f''(0) = 0, f(l) = 0, f''(l) = 0.$$

利用以上 4 个边界条件确定常数  $C_1 \sim C_4$  后,即可得到集中荷载作用下简支箱梁剪力滞附加挠度的计算公式:

$$f = \frac{2Pk_1 l^3}{\pi^2} \left( \alpha_1 \sin \frac{\pi z}{l} - \alpha_2 \sin \frac{3\pi z}{l} + \alpha_3 \sin \frac{5\pi z}{l} - \alpha_4 \sin \frac{7\pi z}{l} \right). \quad (22)$$

由材料力学可知简支箱梁集中荷载作用下的挠度计算公式,将初等梁挠度和剪力滞附加挠度计算公式代入式(3)可得箱梁截面任一点处的纵向应力表达式,即

$$\sigma(x, y, z) = \frac{Pyz}{2I} + 2Ek_1 Pl \omega_\xi(x, y) \left( \alpha_1 \sin \frac{\pi z}{l} - 9\alpha_2 \sin \frac{3\pi z}{l} + 25\alpha_3 \sin \frac{5\pi z}{l} - 49\alpha_4 \sin \frac{7\pi z}{l} \right) \left( 0 \leq z \leq \frac{l}{2} \right). \quad (23)$$

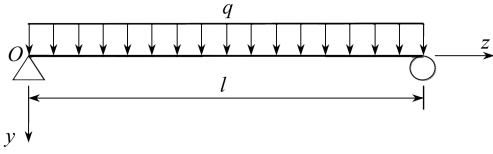


图 3 简支箱梁受均布荷载作用

Fig. 3 A simply supported box girder under a uniform load

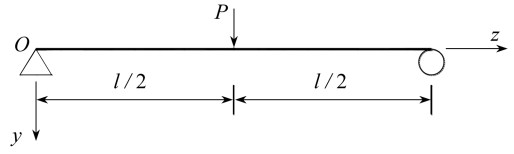


图 4 简支箱梁受集中荷载作用

Fig. 4 A simply supported box girder under a concentrated load

### 3 算例分析

以文献[17]跨度 50 m 的简支混凝土斜腹板箱梁为例,截面尺寸、计算点位置见图 5,材料参数为  $E = 3.1 \times 10^4$  MPa, Poisson(泊松)比  $\mu = 1/6$ . 满跨均布荷载  $2q = 2 \times 1$  kN/m; 跨中集中荷载  $2P = 2 \times 10$  kN.

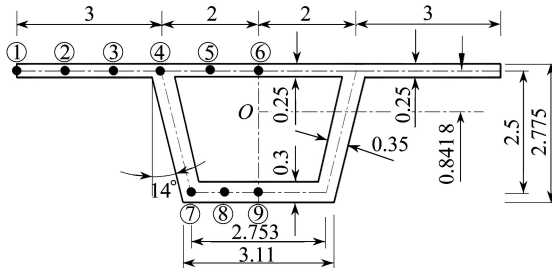


图 5 截面尺寸及计算点(单位: m)

Fig. 5 The cross section sizes and calculation points (unit: m)

利用本文导出的集中荷载和均布荷载作用下简支箱梁的纵向应力计算公式,分别计算两种荷载作用下跨中截面计算点的纵向应力,与文献[17]给出的样条函数解一并列于表 1,以便对比(表中,误差比=(本文解-样条函数解)/样条函数解 $\times 100\%$ ).

表 1 跨中截面计算点纵向应力对比(单位:kPa)

Table 1 Comparison of longitudinal stresses at the span center section (unit: kPa)

calculation point	concentrated load			uniform load		
	spline function solution <sup>[17]</sup> $\sigma_p$ /kPa	present solution $\sigma_p$ /kPa	error ratio $\varepsilon$ /%	spline function solution <sup>[17]</sup> $\sigma_q$ /kPa	present solution $\sigma_q$ /kPa	error ratio $\varepsilon$ /%
①	-33.507 0	-38.225	14.08	-98.448 5	-98.550	1.65
②	-35.334 0	-38.652	9.39	-98.784 2	-98.837	1.27
③	-37.880 5	-39.819	5.12	-99.701 0	-99.621	0.75
④	-42.265 5	-41.414	-2.01	-100.953 5	-100.693	-0.03
⑤	-39.320 5	-39.159	-0.41	-99.182 0	-99.178	-0.45
⑥	-38.526 0	-38.225	-0.78	-98.448 5	-98.550	-0.72
⑦	78.960 5	81.557	3.29	198.860 9	198.348	0.80
⑧	75.775 7	77.137	1.80	195.371 8	195.364	-0.17
⑨	75.087 0	75.296	0.28	193.926 6	194.127	-0.66

表 1 给出了简支箱梁受集中荷载和均布荷载作用下跨中截面计算点的纵向应力对比,可以看出本文方法计算结果与样条函数计算结果吻合良好,从而验证了该方法的正确性。

采用图 5 所示的箱梁截面尺寸、跨度及荷载工况,利用本文附加挠度公式和材料力学初等

梁挠度公式分别计算集中荷载和均布荷载作用下相应截面的挠度解,并绘制挠度分布曲线,如图6、图7所示,并将两种荷载作用下跨中截面的挠度列于表2,以便对比。保持箱梁截面尺寸不变,调整跨长并获得相应的高跨比和宽跨比参数,应用本文公式计算相应参数下跨中截面的附加挠度,并绘制附加挠度随参数变化的分布曲线,如图8、图9所示。

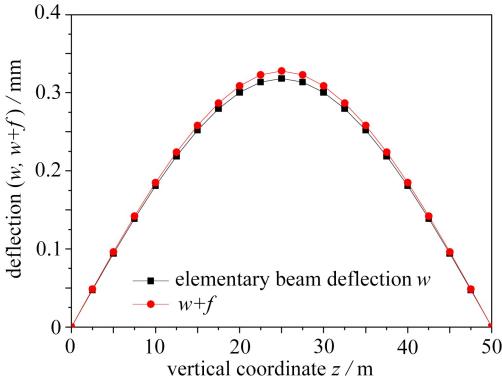


图6 集中荷载作用下的挠曲线

Fig. 6 The deflection curve under the concentrated load

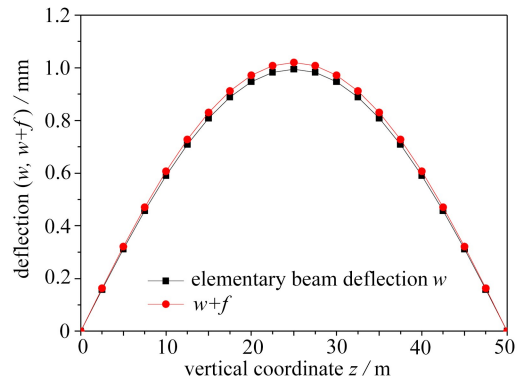


图7 均布荷载作用下的挠曲线

Fig. 7 The deflection curve under the uniform load

表2 跨中截面挠度对比

Table 2 Comparison of additional deflections at the span center section

location	uniform load			concentrated load		
	$w / \text{mm}$	$f / \text{mm}$	$(f/w) / \%$	$w / \text{mm}$	$f / \text{mm}$	$(f/w) / \%$
the span center section	0.994	0.025 6	2.57	0.318	0.009 6	3.03

图6、图7给出了集中荷载和均布荷载作用下箱梁挠度沿纵向的分布曲线,可以看出:两种荷载作用下考虑剪力滞效应的总挠度分布呈曲线分布;附加挠度的大小由跨中向两侧支点递减。由表2可以看出,均布荷载和集中荷载作用下跨中截面的附加挠度分别占初等梁挠度的2.57%和3.03%。

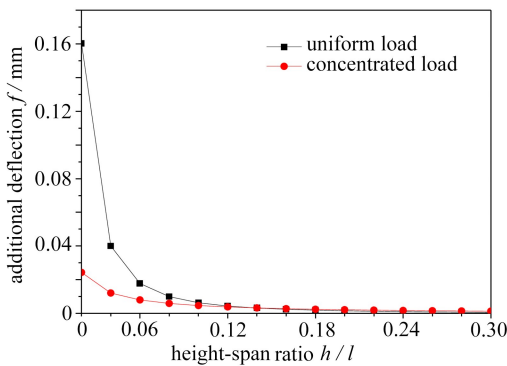


图8 附加挠度随高跨比的变化曲线

Fig. 8 The variation curve of the additional deflection with the height-span ratio

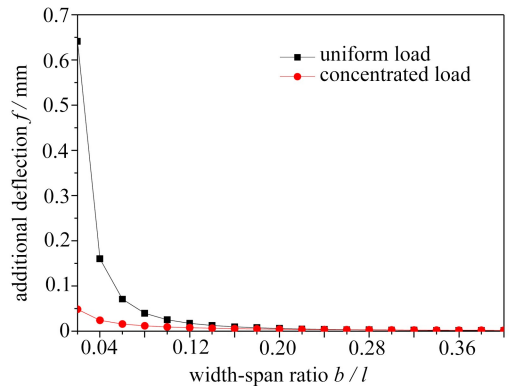


图9 附加挠度随宽跨比的变化曲线

Fig. 9 The variation curve of the additional deflection with the width-span ratio

图8、图9给出了均布荷载和集中荷载作用下高跨比和宽跨比对箱梁附加挠度的影响曲线,可以看出:随着高跨比和宽跨比的增大,相应跨中截面的剪力滞附加挠度逐渐减小;显然,宽跨比对箱梁跨中截面附加挠度的影响远大于高跨比的影响。当高跨比在0.02~0.06范围内,

相应箱梁跨中截面的附加挠度降低幅度较大,在大于 0.06 的范围内附加挠度的降低幅度较小,基本趋于零;同样,宽跨比在 0.02~0.04 范围内,相应箱梁跨中截面的附加挠度降低幅度较大,在大于 0.04 的范围内附加挠度的降低幅度较小,基本趋于零。

## 4 结 论

1) 本文选取剪力滞效应引起的附加挠度为广义位移,将箱梁的剪力滞变形状态从初等梁挠曲变形状态中分离出来作为一种独立的变形状态分析,应用能量变分法建立了剪力滞效应分析理论.算例分析表明,本文应力计算结果与样条函数计算结果吻合良好,验证了本文方法的正确性。

2) 考虑剪力滞效应后的箱梁总挠度沿纵向呈曲线分布;附加挠度的大小由跨中向两侧支点点递减.针对本文算例而言,均布荷载和集中荷载作用下跨中截面的剪力滞附加挠度分别占初等梁挠度的 2.57% 和 3.03%。

3) 随着高跨比和宽跨比的增大,相应箱梁跨中截面的附加挠度逐渐减小.当取同一高跨比和宽跨比时,宽跨比对附加挠度的影响明显大于高跨比对附加挠度的影响。

**致谢** 本文作者衷心感谢兰州交通大学青年基金(2018017)对本文的资助。

## 参考文献(References):

- [1] 张元海, 胡玉茹, 林丽霞. 基于修正翘曲位移模式的薄壁箱梁剪力滞效应分析[J]. 土木工程学报, 2015, **48**(6): 44-45. (ZHANG Yuanhai, HU Yuru, LIN Lixia. Analysis on shear lag effect of thin-walled box girders based on a modified warping displacement mode[J]. *Journal of Civil Engineering*, 2015, **48**(6): 44-45. (in Chinese))
- [2] VISNJIC G, NOZAK D, KOSEL F, et al. Shear-lag influence on maximum specific bending stiffness and strength of composite I-beam wing spar[J]. *Journal of Aerospace Engineering*, 2011, **225**(5): 501-511.
- [3] FABRIZIO G, GIANLUCA R, GRAZIANO L. Partial interaction analysis with shear-lag effects of composite bridges: a finite element implementation for design applications[J]. *Advanced Steel Construction*, 2011, **7**(1): 1-16.
- [4] ZHANG Y H. Improved finite-segment method for analyzing shear lag effect in thin-walled box girders[J]. *Journal of Structural Engineering*, 2012, **138**(10): 1279-1284.
- [5] ZHOU S J. Finite beam element considering shear-lag effect in box girder[J]. *Journal of Engineering Machinery*, 2010, **136**(9): 1115-1122.
- [6] REISSNER E. Analysis of shear lag in box beams by the principle of the minimum potential energy[J]. *Quarterly of Applied Mathematics*, 1946, **4**(3): 268-278.
- [7] 钱寅泉, 倪元增. 单室箱桥的剪力滞分析[J]. 中国公路学报, 1989, **2**(2): 28-37. (QIAN Yinquan, NI Yuanzeng. Shear lag analysis of single room box girder[J]. *Chinese Journal of Highway and Transport*, 1989, **2**(2): 28-37. (in Chinese))
- [8] 倪元增, 钱寅泉. 箱梁剪力滞计算的翘曲函数法[J]. 铁道学报, 1990, **12**(2): 57-70. (NI Yuanzeng, QIAN Yinquan. Warping function method for shear lag calculation of box girder [J]. *Journal of the China Railway Society*, 1990, **12**(2): 57-70. (in Chinese))
- [9] 周茂定, 李丽园, 张元海. 薄壁箱梁的剪力滞翘曲位移函数研究[J]. 中国公路学报, 2015, **28**(6): 67-73. (ZHOU Maoding, LI Liyuan, ZHANG Yuanhai. Research on shear-lag displacement function of thin-walled boxed girders[J]. *China Journal of Highway and Transport*, 2015,



- 28(6): 67-73.(in Chinese))
- [10] 谢旭, 黄剑源. 薄壁箱型梁剪力滞效应分析的刚度法[J]. 工程力学, 1995, 12(2): 95-101.(XIE Xu, HUANG Jianyuan. Stiffness method for analysis of shear-lag effect on thin-walled boxed girder[J]. *Engineering Mechanics*, 1995, 12(2): 95-101.(in Chinese))
- [11] 冀伟, 吴焕庆, 刘世忠. 附加轴向位移对薄壁箱梁翼板应力的影响[J]. 公路交通科技, 2013, 30(9): 69-73.(JI Wei, WU Huanqing, LIU Shizhong. Influence of additional axial displacement on flange stress of thin-walled box girder[J]. *Highway Traffic Science and Technology*, 2013, 30(9): 69-73.(in Chinese))
- [12] 韦成龙, 曾庆元, 刘小燕. 薄壁箱梁剪力滞分析的多参数翘曲位移函数及其有限元法[J]. 铁道学报, 2000, 22(5): 60-64.(WEI Chenglong, ZENG Qingyuan, LIU Xiaoyan. Warping displacement function and finite element method for calculation of shear lag effect in box girder[J]. *Journal of the China Railway Society*, 2000, 22(5): 60-64.(in Chinese))
- [13] 周坚. 用余能原理解槽型宽梁的剪力滞问题[J]. 北京建筑工程学院学报, 1985, 7(5): 42-46.(ZHOU Jian. On the shear lag problem of the groove type wide beam with the complementary energy principle[J]. *Journal of Beijing Institute of Civil Engineering and Architecture*, 1985, 7(5): 42-46.(in Chinese))
- [14] 周坚, 涂令康. 再论槽型宽梁的剪力滞[J]. 北京建筑工程学院学报, 1991, 11(2): 44-53.(ZHOU Jian, TU Linggang. Further discussion on the shear hysteresis in the channel-shaped beam[J]. *Journal of Beijing Institute of Civil Engineering and Architecture*, 1991, 11(2): 44-53.(in Chinese))
- [15] 张慧, 张玉元, 张元海, 等. 单箱双室简支箱梁剪切变形及剪力滞双重效应分析[J]. 应用数学和力学, 2016, 37(8): 791-803.(ZHANG Hui, ZHANG Yuyuan, ZHANG Yuanhai, et al. Analysis on shear deformation and shear lag effect on twin-cell box girder[J]. *Applied Mathematics and Mechanics*, 2016, 37(8): 791-803.(in Chinese))
- [16] 杨绿峰, 周月娥. 基于挠度的剪力滞系数分析箱梁剪力滞效应[J]. 防灾减灾工程学报, 2013, 33(5): 494-496.(YANG Lüfeng, ZHOU Yuee. Application of deflection-based shear lag coefficient to analysis of shear lag effect of box-girders[J]. *Journal of Disaster Prevent and Mitigation of Engineering*, 2013, 33(5): 494-496.(in Chinese))
- [17] 罗旗帜. 基于能量原理的薄壁箱梁剪力滞理论与试验研究[D]. 博士学位论文. 长沙: 湖南大学, 2005.(LUO Qizhi. Theory and model test studies of the shear lag in thin-walled box girders based on energy principle[D]. PhD Thesis. Changsha: Hunan University, 2005.(in Chinese))

# A Separate Solution Method for Shear Lag Effects in Box Girders and Parameter Analysis

ZHANG Yuyuan, ZHANG Yuanhai, ZHANG Hui

(School of Civil Engineering, Lanzhou Jiaotong University,  
Lanzhou 730070, P.R.China)

**Abstract:** With the additional deflection caused by the shear lag effect as a generalized displacement, the shear lag deformation of the box girder was separated from the elementary beam deflection as an independent state for analysis. The governing differential equations and the corresponding boundary conditions were established with the energy variational method. According to the simply supported boundary conditions, the analytical solutions of the additional deflection and the longitudinal stress were deduced under the concentrated load and the uniform load. The longitudinal stress analysis shows that the stresses calculated with the present method were in good agreement with those with the spline function method, validating the rationality of the present method. The deflection study shows that the additional deflection of shear lag diminishes from the span center section to the 2 ends. The shear lag additional deflection at the span center section reaches respectively 2.57% and 3.03% of the elementary beam deflection of the simply supported box girder under the uniform load and the concentrated load. With the increases of the height-span and width-span ratios, the additional deflection at the span center section of the box girder gradually decreases. In particular, the influence of the width-span ratio on the additional deflection is far greater than that of the height-span ratio.

**Key words:** thin-walled box girder; shear lag warping; elementary beam theory; additional deflection; energy variational method

**Foundation item:** The National Natural Science Foundation of China ( 51468032; 51268029; 51508255 )

---

引用本文/Cite this paper:

张玉元, 张元海, 张慧. 箱形梁剪力滞效应的分离求解方法及参数影响分析[J]. 应用数学和力学, 2018, **39**(11): 1282-1291.

ZHANG Yuyuan, ZHANG Yuanhai, ZHANG Hui. A separate solution method for shear lag effects in box girders and parameter analysis[J]. *Applied Mathematics and Mechanics*, 2018, **39**(11): 1282-1291.