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激光脉冲放大器增益通量耦合系统解*

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摘要： 研究了一个激光脉冲放大器增益通量系统解的问题.首先讨论了较一般的系统, 然后引入一个同伦映射.再利用映射的性质, 引进一个人工参数, 将求解非线性问题转化为求解一系列线性问题.再逐次地求出对应的线性问题的解, 最后得到了原模型解的近似展开式.可以看出, 同伦映射方法是一个解析的方法.它是通过函数的解析运算并用初等函数来表达近似解,其不同于用离散数值运算的数值计算方法.因此通过同伦映射解, 还可以对它继续进行解析运算, 从而可以进行微分和积分等运算来得到与激光脉冲放大器增益通量相关的其他物理量的性态.

关　键　词： 激光； 放大器； 非线性

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引　　言

激光脉冲放大是近代物理的一个重要研究对象.“神 Ⅱ” 激光放大装置就是典型的课题[1].在激光脉冲放大的研究中, 需要了解相关物理量的性态,譬如,能量增益、能量密度、增益通量、光子数密度、脉冲波形、瞬时功率增益等.目前,对无损耗的激光放大器已经有很多研究,然而,对有损耗的情形还在进一步探索中[2-4].包括建立系统的模型未必可用显式解析解来表述非线性方程模型,而通常是用数值模拟方法来对它进行研究.这就意味着对相应的物理量不能继续进行解析运算,具有一定的局限性.本文是从一类较广泛的激光脉冲放大器模型着手,引入了一个简单有效的广义泛函同伦分析方法,得到了近似解析解,扩大了进一步对激光脉冲放大器相关量的研究途径.

近年来, 许多学者已经讨论了非线性相关问题,对近似方法作了进一步发展、改进和拓广[5-7], 包括边界层法、多尺度法、平均法及匹配渐近展开法等.笔者利用渐近方法也研究了一类非线性问题[8-21].由于同伦分析方法是一个解析求解方法, 同时它也可利用泛函分析映射的理论, 对相应的模型能较简单地直接求出对应模型的广义解;另外, 同伦分析方法不但能讨论奇摄动问题的模型, 而且也能讨论一类非小扰动模型, 然而边界层法、多尺度法、平均法、匹配渐近展开法等方法却不能讨论该类模型.本文就是利用一个简单而特殊的同伦分析[22-23]、

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偏微分方程的首次积分[24-25]等方法和理论来讨论一类非小扰动的激光脉冲放大器模型.

1 激光脉冲放大器模型

本文讨论如下一个含损耗的激光脉冲放大器增益通量耦合系统[2,4]:

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}+\gamma u=\sigma cuv-f(u),\tag{1}$$

$$\frac{\partial v}{\partial t}+\delta v=-\sigma cuv-g(v),\tag{2}$$

$$u|_{t=0}=h_1(x),\tag{3}$$

$$v|_{t=0}=h_2(x),\tag{4}$$

其中, u 为光子数密度, v 为反转粒子数密度, c 为光子的传播速度, σ 为受激光辐射的截面, γ,δ 分别为光子数、反转粒子数在传输过程中的损耗, 扰动函数 $f(u)$, $g(v)$ 分别为其他因素使得光子数、反转粒子数在传输过程中的非线性损耗, $h_i(x)\ (i=1,2)$ 为耦合系统的初值.不妨设 f,g 和 $h_i(x)\ (i=1,2)$ 为各自变元的充分光滑函数.

2 激光脉冲放大器系统的解

由于耦合系统模型(1)~(4)一般不具有初等函数的精确解, 为此本文采用特殊的方法来寻求耦合系统的近似解.设

$$u=\sum_{i=0}^{\infty}u_i(x,t)p^i,\ v=\sum_{i=0}^{\infty}v_i(x,t)p^i,\qquad (x,t)\in\mathbf{R}\times[0,\infty),\tag{5}$$

其中 $p\in[0,1]$ 为人工参数[22-23].

首先引入一组同伦映射 $H_i[u,v]\ (i=1,2)$, $R^2\times I\to\mathbf{R}$:

$$H_1[u,v]=L_1[u]-L_1[u_0]+p(L_1[u_0]-\sigma cuv+f(u)),\tag{6}$$

$$H_2[u,v]=L_2[v]-L_2[v_0]+p(L_2[v_0]-\sigma cuv+g(v)),\tag{7}$$

其中线性算子 $L_i[u,v]\ (i=1,2)$ 为

$$L_1[u]=\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}+\gamma u,\ L_2[v]=\frac{\partial v}{\partial t}+\delta v.$$

由式(6)、(7), 将式(4)代入 $H_1(u,p)=0$, $H_2(v,p)=0$, 按 p 的幂展开非线性项, 合并方程 $H_1(u,p)=0$, $H_2(v,p)=0$ 关于 p 的同次幂系数并等于0.由 p^0 的系数可得

$$L_1[u_0]\equiv\frac{\partial u_0}{\partial t}+c\frac{\partial u_0}{\partial x}+\gamma u_0=0,\tag{8}$$

$$L_2[v_0]\equiv\frac{\partial v_0}{\partial t}+\delta v_0=0.\tag{9}$$

偏微分方程(8)的特征方程为

$$\frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{c}=\frac{\mathrm{d}u_0}{-\gamma u_0}.\tag{10}$$

由系统(10), 不难得知微分方程具有如下两个独立的首次积分:

$$x-ct=C_0,\ u_0=D_0\exp(-\gamma t),$$

其中 C_0, D_0 为任意函数.于是由偏微分系统初值问题(8)、(3)具有解

$$u_0(x,t)=h_1(x-ct)\exp(-\gamma t).\tag{11}$$

显然，方程(8)在初始条件(4)下的解为

$$v_0(x,t)=h_2(x)\exp(-\delta t). \tag{12}$$

由式(6)、(7)，将式(4)代入 $H_1(u,p)=0, H_2(v,p)=0$，按 p 的幂展开非线性项，合并方程 $H_1(u,p)=0, H_2(v,p)=0$ 关于 p 的同次幂系数并等于0.由 p^1 的系数可得

$$L_1[u_1]=\sigma c u_0 v_0-f(u_0), \tag{13}$$

$$L_2[v_1]=\sigma c u_0 v_0-g(v_0). \tag{14}$$

偏微分方程(13)的特征方程为

$$\frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{c}=\frac{\mathrm{d}u_1}{-\gamma u_1+\sigma c u_0 v_0-f(u_0)}. \tag{15}$$

由系统(15)，不难得出微分方程具有如下两个独立的首次积分：

$$x-ct=C_1,$$

$$u_1(x,t)=D_1\exp(-\gamma t)+\int_0^t[\sigma c u_0(x,t_1)v_0(x,t_1)-f(u_0(x,t_1))]\exp(-\gamma(t-t_1))\mathrm{d}t_1,$$

其中 C_1, D_1 为任意函数，u_0, v_0 分别由式(11)、(12)表示.于是偏微分方程(13)在零初值条件下具有解

$$u_1(x,t)=\int_0^t[\sigma c u_0(x,t_1)v_0(x,t_1)-f(u_0(x,t_1))]\exp(-\gamma(t-t_1))\mathrm{d}t_1. \tag{16}$$

而方程(14)在零初始条件下的解为

$$v_1(x,t)=\int_0^t[-\sigma c u_0(x,t_1)v_0(x,t_1)-g(v_0(x,t_1))]\exp(-\delta(t-t_1))\mathrm{d}t_1. \tag{17}$$

由式(6)、(7)，将式(4)代入 $H_1(u,p)=0, H_2(v,p)=0$，按 p 的幂展开非线性项.由 p^2 的系数可得

$$L_1[u_2]=\sigma c(u_0 v_1+u_1 v_0)-f_u(u_0)u_1, \tag{18}$$

$$L_1[v_2]=\sigma c(u_0 v_1+u_1 v_0)-g_v(v_0)v_1. \tag{19}$$

偏微分方程(18)的特征方程为

$$\frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{c}=\frac{\mathrm{d}u_1}{-\gamma u_1+\sigma c(u_0 v_1+u_1 v_0)-f_u(u_0)u_1}. \tag{20}$$

由系统(20)，不难得出微分方程具有如下两个独立的首次积分：

$$x-ct=C_2,$$

$$u_2(x,t)=D_2\exp(-\gamma t)+\int_0^t[\sigma c(u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-f_u(u_0(x,t_1))u_1(x,t_1)]\exp(-\gamma(t-t_1))\mathrm{d}t_1,$$

其中 C_2, D_2 为任意函数，u_0, v_0, u_1, v_1 分别由式(11)、(12)和式(16)、(17)表示.于是偏微分方程(18)在零初值条件下具有解

$$u_2(x,t)=\int_0^t[-\gamma u_1(x,t_1)+\sigma c(u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-f_u(u_0(x,t_1))u_1(x,t_1)]\exp(-\gamma(t-t_1))\mathrm{d}t_1. \tag{21}$$

而方程(19)在零初始条件下的解为

$$v_2(x,t)=\int_0^t[-\sigma c(u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-$$

$$g_v(v_0(x,t_1)v_1(x,t_1))]\exp(-\delta(t-t_1))\mathrm{d}t_1. \tag{22}$$

同样地，将式(4)代入 $H_1(u,p)=0,H_2(v,p)=0$，按 p 的幂展开非线性项.由 $p^i(i=2,3,\cdots)$ 的系数为 0 有

$$\frac{\partial u_i}{\partial t}+c\frac{\partial u_i}{\partial x}+\gamma u_i=F_i,\qquad i=3,4,\cdots, \tag{23}$$

$$\frac{\partial v_i}{\partial t}+\delta v_i=G_i,\qquad i=3,4,\cdots, \tag{24}$$

$$u_i\big|_{t=0}=v_i\big|_{t=0}=0,\qquad i=3,4,\cdots, \tag{25}$$

其中 $F_i,G_i(i=3,4,\cdots)$ 为逐次已知的函数：

$$F_i=\frac{1}{i!}\left[\frac{\partial^i}{\partial p^i}\left[\sigma c\left(\sum_{j=0}^{\infty}u_j(x,t)p^j\right)\left(\sum_{j=0}^{\infty}v_j(x,t)p^j\right)-f\left(\sum_{j=0}^{\infty}u_j(x,t)p^j\right)\right]\right]_{p=0},$$
$$i=3,4,\cdots,$$

$$G_i=\frac{1}{i!}\left[\frac{\partial^i}{\partial p^i}\left[-\sigma c\left(\sum_{j=0}^{\infty}u_j(x,t)p^j\right)\left(\sum_{j=0}^{\infty}v_j(x,t)p^j\right)-g\left(\sum_{j=0}^{\infty}u_j(x,t)p^j\right)\right]\right]_{p=0},$$
$$i=3,4,\cdots.$$

于是,可得线性系统(23)~(25)的解为

$$u_i(t,x,t)=\int_0^t F_i(x,t_2)\exp(-\gamma(t-t_1))\mathrm{d}t_1,\qquad i=3,4,\cdots, \tag{26}$$

$$v_i(x,t)=-\int_0^t G_i(x,t_2)\exp(-\delta(t-t_1))\mathrm{d}t_1,\qquad i=3,4,\cdots. \tag{27}$$

由式(11)、(12)、(16)、(17)、(21)、(22)、(26)、(27)，将求得的 $u_i,v_i(i=0,1,\cdots)$ 代入式(5)，并令 $p=1$，便得到

$$\begin{aligned}u(x,t)={}&h_1(x-ct)\exp(-\gamma t)+\\&\int_0^t[\sigma c(u_0(x,t_1)v_0(x,t_1)+u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-\\&(f(u_0(x,t_1))+f_u(u_0(x,t_1)u_1(x,t_1)))]\exp(-\gamma(t-t_1))\mathrm{d}t_1+\\&\sum_{i=3}^{\infty}\left[\int_0^t F_i(x,t_2)\exp(-\gamma(t-t_1))\mathrm{d}t_1\right],\end{aligned} \tag{28}$$

$$\begin{aligned}v(x,t)={}&h_2(x)\exp(-\delta t)+\\&\int_0^t[-\sigma c(u_0(x,t_1)v_0(x,t_1)+u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-\\&(g(v_0(x,t_1))+g_v(v_0(x,t_1)v_1(x,t_1)))]\exp(-\delta(t-t_1))\mathrm{d}t_1+\\&\sum_{i=3}^{\infty}\left[\int_0^t G_j(x,t_2)\exp(-\delta(t-t_1))\mathrm{d}t_1\right].\end{aligned} \tag{29}$$

由泛函分析不动点理论[24-25]，函数 $f,g,h_i(i=1,2)$ 在对应的定义域内为光滑有界函数的条件下，可以证明由式(28)、(29)决定的 $(u(x,t),v(x,t))$ 是含损耗的激光脉冲放大器增益通量耦合系统(1)~(4)在对应定义域内为一致收敛的解.因而

$$\begin{aligned}U_n(x,t)={}&h_1(x-ct)\exp(-\gamma t)+\\&\int_0^t[\sigma c(u_0(x,t_1)v_0(x,t_1)+u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-\\&(f(u_0(x,t_1))+f_u(u_0(x,t_1)u_1(x,t_1)))]\exp(-\gamma(t-t_1))\mathrm{d}t_1+\end{aligned}$$

$$\sum_{i=3}^{n}\left[\int_0^t F_i(x,t_2)\exp(-\gamma(t-t_1))\mathrm{d}t_1\right], \tag{30}$$

$$V_n(x,t)=h_2(x)\exp(-\delta t)+$$
$$\int_0^t[-\sigma c(u_0(x,t_1)v_0(x,t_1)+u_0(x,t_1)v_1(x,t_1)+u_1(x,t_1)v_0(x,t_1))-$$
$$(g(v_0(x,t_1))+g_v(v_0(x,t_1)v_1(x,t_1)))]\exp(-\delta(t-t_1))\mathrm{d}t_1+$$
$$\sum_{i=3}^{n}\left[\int_0^t G_j(x,t_2)\exp(-\delta(t-t_1))\mathrm{d}t_1\right] \tag{31}$$

为激光脉冲放大器增益通量耦合系统(1)~(4)的 n 次近似解.

3 放大器模型的例

作为一个由激光脉冲放大器系统求近似解简单的例子，对于系统(1)~(4)，不妨取无量纲扰动函数和初始值为：$f(u)=du^2$，$g(v)=0$，$h_1(x)=x$，$h_2(x)=x$，其中 d 为常数.此时式(1)~(4)为如下无量纲系统：

$$\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}+\gamma u=\sigma cuv-du^2, \tag{32}$$

$$\frac{\partial v}{\partial t}+\delta v=-\sigma cuv, \tag{33}$$

$$u|_{t=0}=x,\ v|_{t=0}=x. \tag{34}$$

利用本文所用的方法，设

$$u=\sum_{i=0}^{\infty}u_i(x,t)p^i,\ v=\sum_{i=0}^{\infty}v_i(x,t)p^i.$$

由式(11)、(12)，有

$$u_0(x,t)=(x-ct)\exp(-\gamma t),\ v_0(x,t)=x\exp(-\delta t). \tag{35}$$

由式(16)、(17)，有

$$u_1(x,t)=\frac{\sigma c}{\delta^2}x\exp(-\gamma t)((\delta x+c)-(\delta x-c-c\delta t))\exp(-\delta t)+$$
$$\frac{d}{\gamma}\left(x^2-\frac{2c^2}{\gamma^2}\left(\frac{\gamma}{c}(x-ct)-1\right)\right)\exp(-2\gamma t)-$$
$$\frac{d}{\gamma}\left(x^2+\frac{2c^2}{\gamma^2}\left(\frac{\gamma}{c}x-1\right)\right)\exp\left(-\frac{\gamma}{c}t\right), \tag{36}$$

$$v_1(x,t)=\frac{\sigma c}{\gamma^2}x\exp(-\delta t)((\gamma x+c)-(\gamma x-c-c\gamma t))\exp(-\gamma t). \tag{37}$$

由式(21)、(22)，有

$$u_2(x,t)=\int_0^t[(-\gamma(x-ct_1)+$$
$$\sigma c(x-ct_1)v_1(x,t_1))\exp(-\gamma t_1)+x\exp(-\delta t_1)u_1(x,t_1)-$$
$$2d(x-ct)\exp(-\gamma t_1)u_1(x,t_1)]\exp(-\gamma(t-t_1))\mathrm{d}t_1, \tag{38}$$

$$v_2(x,t)=\int_0^t[-\sigma(x-ct_1)\exp(-\gamma t_2)v_1(x,t_1)+$$
$$u_1(x,t_1)x\exp(-\delta t)]\exp(-\delta(t-t_1))\mathrm{d}t_1, \tag{39}$$

在式(38)、(39)中 u_1,v_1 由式(36)、(37)表示.

由式(35)~(39),可得到激光脉冲放大器无量纲系统(32)~(34)在有界区域内的二次近似解 $(U_2(x,t),V_2(x,t))$:

$$
\begin{aligned}
U_2(x,t) = {} & (x-ct)\exp(-\gamma t) + \\
& \frac{\sigma c}{\delta^2} x\exp(-\gamma t)((\delta x + c) - (\delta x - c - c\delta t))\exp(-\delta t) + \\
& \frac{d}{\gamma}\left(x^2 - \frac{2c^2}{\gamma^2}\left(\frac{\gamma}{c}(x-ct) - 1\right)\right)\exp(-2\gamma t) - \\
& \frac{d}{\gamma}\left(x^2 + \frac{2c^2}{\gamma^2}\left(\frac{\gamma}{c}x - 1\right)\right)\exp\left(-\frac{\gamma}{c}t\right) + \\
& \int_0^t [(-\gamma(x-ct_1) + \sigma c(x-ct_1)v_1(x,t_1))\exp(-\gamma t_1) + \\
& x\exp(-\delta t_1)u_1(x,t_1) - 2d(x-ct)\exp(-\gamma t_1)u_1(x,t_1)]\exp(-\gamma(t-t_1))\mathrm{d}t_1,
\end{aligned}
$$

$$
\begin{aligned}
V_2(x,t) = {} & x\exp(-\delta t) + \frac{\sigma c}{\gamma^2} x\exp(-\delta t)((\gamma x + c) - (\gamma x - c - c\gamma t))\exp(-\gamma t) + \\
& \int_0^t (-\sigma(x-ct_1)\exp(-\gamma t_2)v_1(x,t_1) + \\
& u_1(x,t_1)x\exp(-\delta t))\exp(-\delta(t-t_1))\mathrm{d}t_1,
\end{aligned}
$$

其中 u_1,v_1 由式(36)、(37)表示.

由式(30)、(31),还可继续进行运算,得到激光脉冲放大器无量纲系统(32)~(34)在有界区域内的更高次近似解 $(U_n(x,t),V_n(x,t))$ $(n=3,4,\cdots)$.

4 结　论

对激光脉冲放大器的相关物理量,例如:能量增益、脉冲波形、光子数密度、功率平衡等的研究,需要在相应模型下对放大输运系统的研究.本文所述的同伦分析映射就是一个有效求出其渐近解析解的方法.

用同伦分析映射方法求得近似解析解,它还可进行解析运算.因此,利用其近似解析解还可以研究更多的相应物理量及其性态.例如,在本文中得到的有耗损情形下的激光放大器模型的光子数密度 u 和反转粒子数密度 v 的近似式,进一步对它们进行微分、积分、卷积等解析运算,从而可得到相应模型的激光脉冲能量密度和光子数密度、瞬时功率增益及激光增益通量等物理量的瞬时值及其近似曲线、曲面图形等性态,使得人们对激光放大器模型的性状有更深入的了解.

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Solutions to the Gain Flux Coupling System of Laser Pulse Amplifiers

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Abstract: The solutions to the gain flux coupling system of laser pulse amplifiers were studied. Firstly, the system of the general model was discussed; secondly, the homotopic mapping was used and an artificial parameter was introduced with the property of the mapping, to transform the nonlinear problem to a series of linear problems, which were solved one by one. Then the approximate expressions of the solutions to the corresponding model were obtained. The expansion of solutions with the homotopic mapping method is analytic, where the analytic operations of the functions are kept and the approximate solutions are expressed with elementary functions, which are different from the numrically computed discrete solutions and can be further analytically computed. Thus the differential and integral operations can be implemented to obtain other physical behaviors of the gain flux for laser pulse amplifiers.

Key words: laser; amplifier; nonlinear

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